Gas-based detectors

Avalanche fluctuations

G.U. Yule (1923), W.H. Furry (1937), R.A. Wijsman (1949) & others

- If the distance between ionisations fluctuates exponentially with a mean of $1/\alpha$ (reciprocal of the Townsend coefficient),

- then, the avalanche size fluctuates (nearly) exponentially:

$$p(n) = \frac{1}{\bar{n}}\left(1 - \frac{1}{\bar{n}}\right)^{n-1}$$

George Udny Yule
(1871-1951)

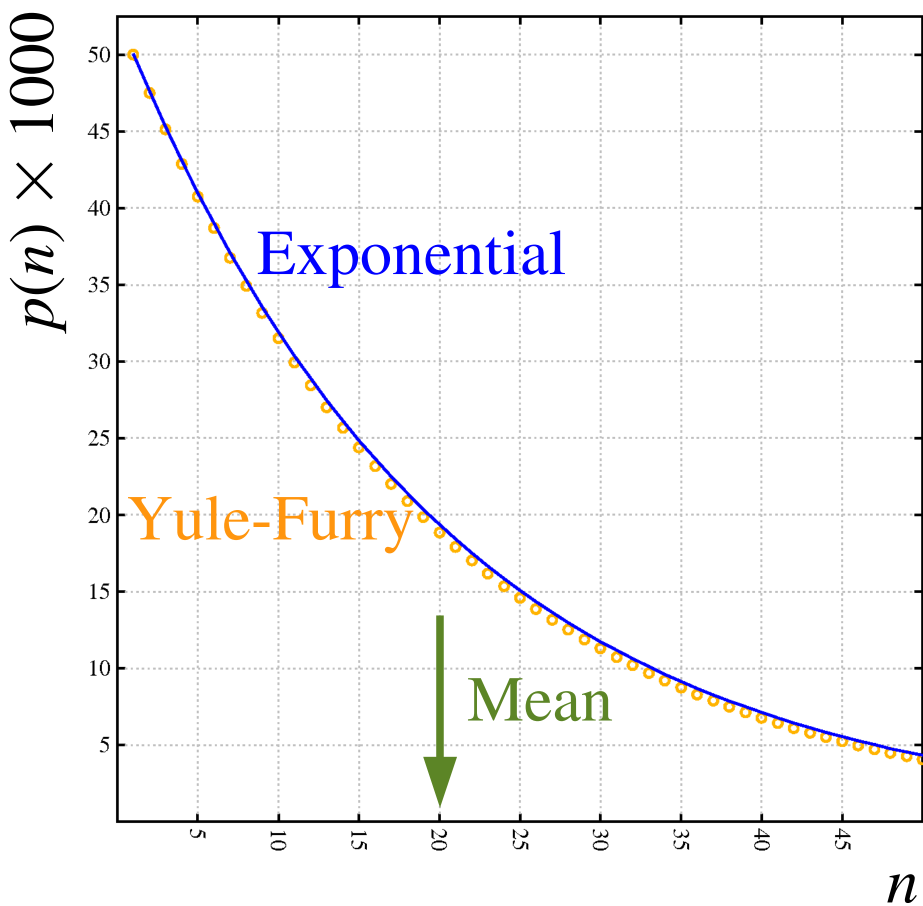
[G. Udny Yule, *A Mathematical Theory of Evolution, based on the Conclusions of Dr. J.C. Willis, F.R.S.*, Phil. Trans. Roy. Soc. London B **213** (1925) 21-87.
W.H. Furry, *On Fluctuation Phenomena in the Passage of High Energy Electrons through Lead*, Phys. Rev. **52** (1937) 569-581.
Robert A. Wijsman, *Breakdown Probability of a Low Pressure Gas Discharge*, Phys. Rev. **75** (1949) 833-838.]

$$f \equiv \sigma^2/\bar{n}^2 \approx 1$$

Statistics Yule-Furry

- Yule-Furry is exponential for large mean avalanche sizes:

$$p(n) = \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1} \approx \frac{e^{-n/\bar{n}}}{\bar{n}-1}$$

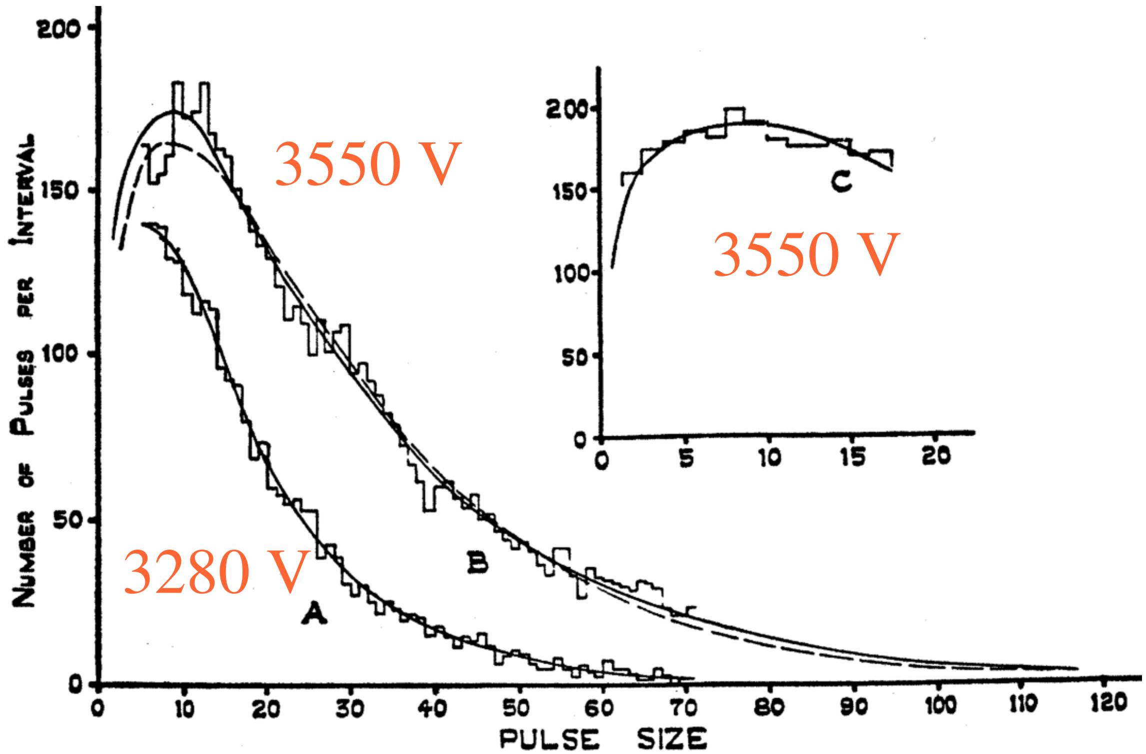


- Mean: $\bar{n}$

  RMS: $\bar{n}\sqrt{1-1/\bar{n}} \approx \bar{n}$

S.C. Curran (1949)

- S.C. Curran *et al*. measured the pulse height distribution in a cylindrical counter (d = 150 µm wire, Ar 50 % CH_4 50 %, p = 670 mbar) at $G \sim 10^4$-10^5:



$$p(n) = \sqrt{n}\,e^{-n}$$

$$f \equiv \left(\frac{\sigma}{\bar{n}}\right)^2 \approx \frac{2}{3}$$

Pólya distribution

- When mathematicians speak of a Pólya distribution, they refer to a negative binomial distribution.

- Avalanche papers mean a Γ distribution:

$$P(g) \propto g^{\theta} \mathrm{e}^{-(1+\theta) g}$$

Note: we sometimes shift θ by one unit !

- and sometimes make reference to a 1923 paper which deals with railway accidents, diseases and flowers.

> Der Tod einer Person infolge Eisenbahnunfalls muß als eine außerordentliche Verschlechterung der Chancen aller Mitreisenden angesehen werden.

- [F. Eggenberger and G. Pólya, Über die Statistik verketteter Vorgänge, Zeitschrift für Angewandte Mathematik und Mechanik **3** (1923) 279-289.]

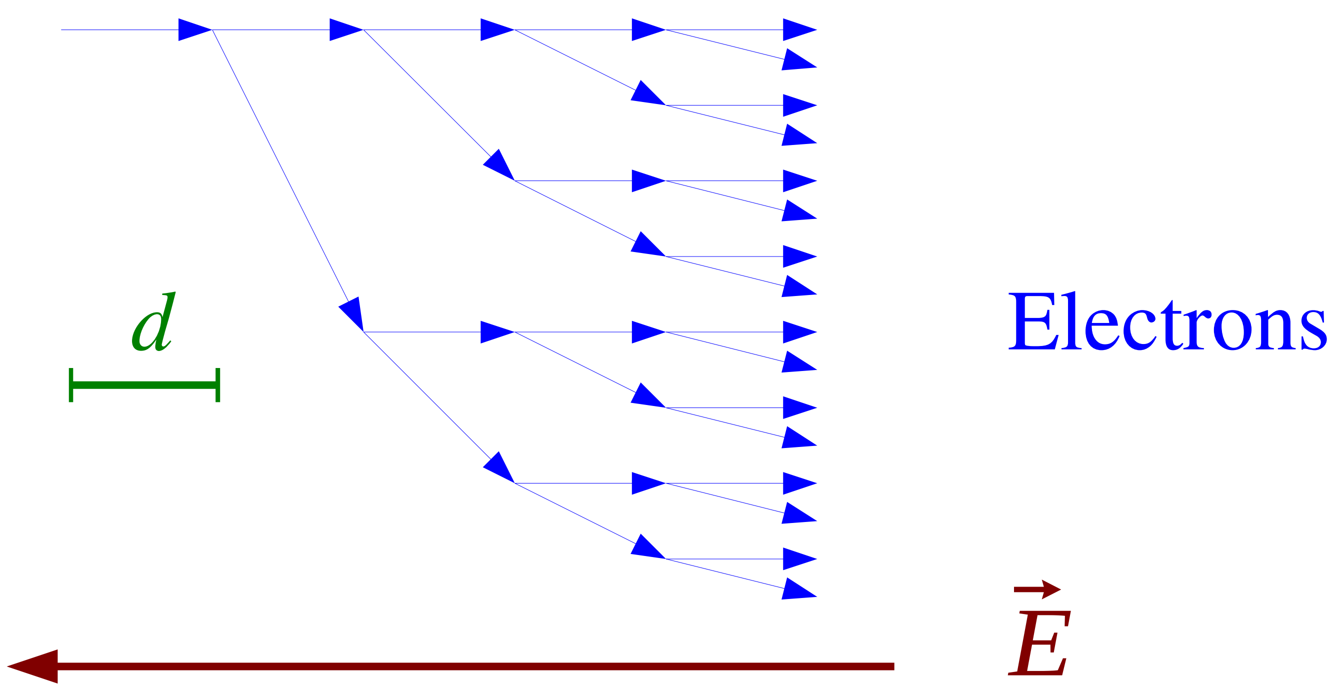
$$f \equiv \sigma^2 / \bar{n}^2$$

The "hump"

- A "rounded" gain distribution ($f < 1$) is beneficial:
  - reduced efficiency loss because small multiplication is not the most probable scenario;
  - reduced probability of large gain and discharge;
  - better energy resolution, better particle identification.

Avalanche size spread – fixed steps

- Imagine an electron *always* creates a secondary after it has traveled *precisely* a distance $d = 1/\alpha$:



- Such an avalanche does not fluctuate: $f = 0$!

Assumptions

- Yule-Furry follows if one assumes:
  - probability to ionise over a distance $\mathrm{d}x$ is $\alpha\,\mathrm{d}x$

    =

    distance between ionisations fluctuates exponentially with mean $1/\alpha$.
  - no history: Townsend coefficient α is constant,
  - no attachment losses.

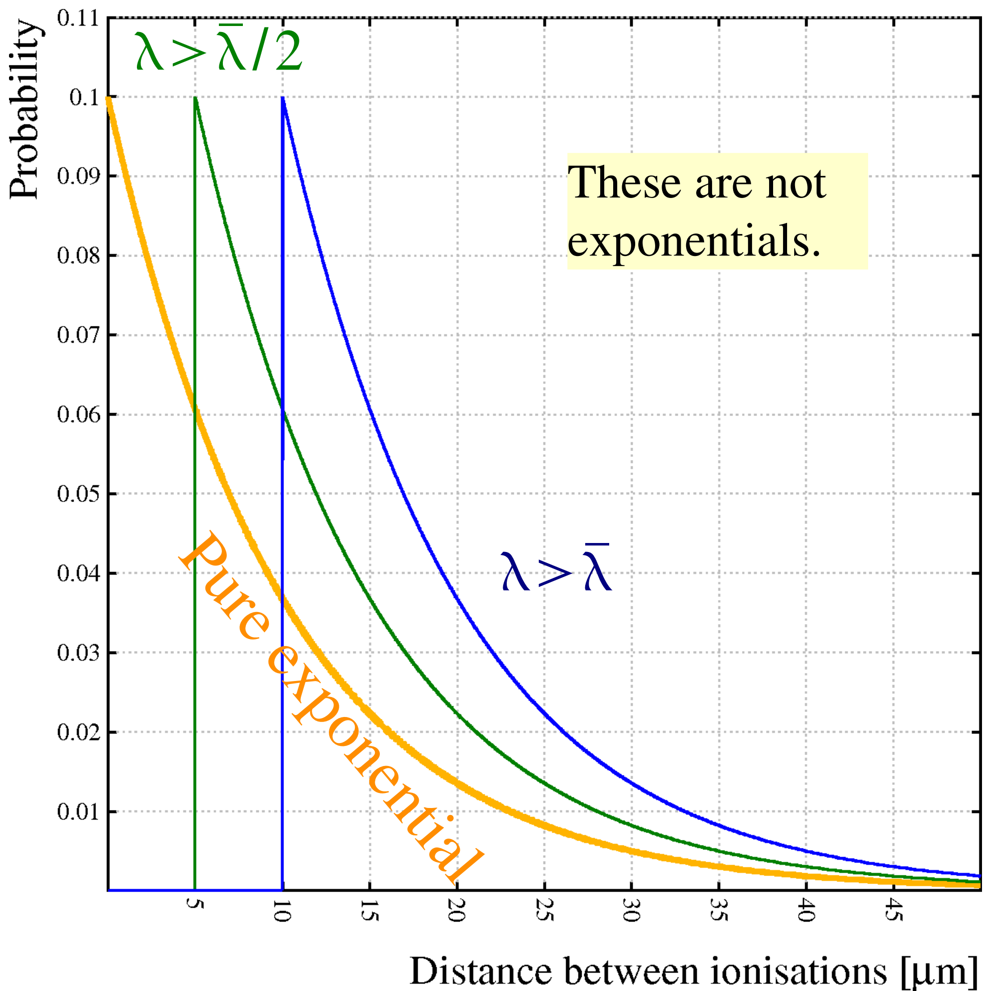
Two schools of thought ...

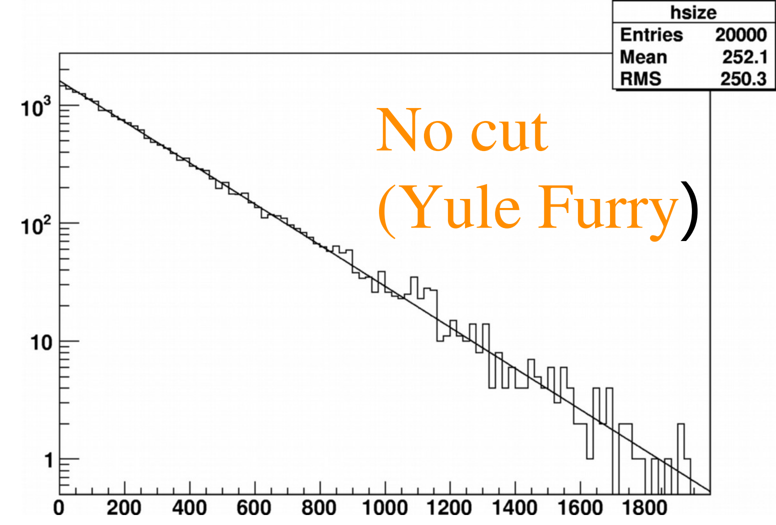
- The distance between ionisations does not simply vary exponentially (e.g. the Raether group).

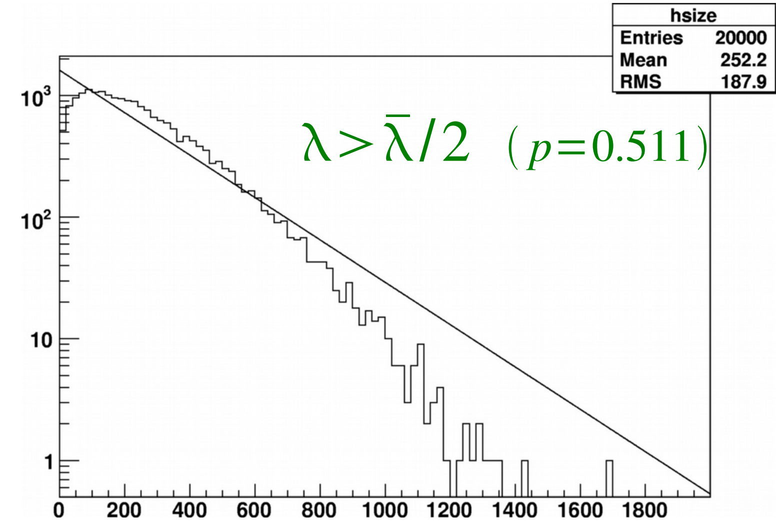
- The Townsend coefficient is not constant (e.g. Byrne, Lansiart & Morucci).

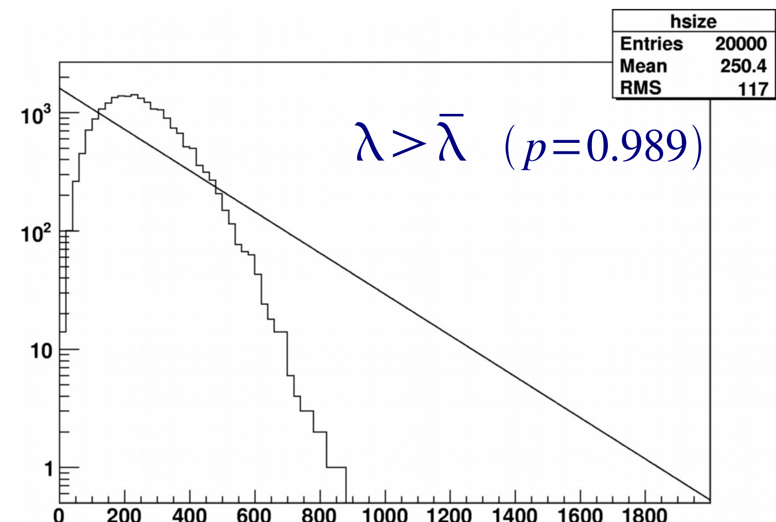
Minimum step length

- Imposing a minimum distance between ionisations adds a hump.









κ – mean / minimum ionisation distance

- When an electron has just ionised, it is not likely to have enough energy left to ionise again straight away: it first has to pick up energy from the *E* field.
- Quantifying:
  - *Mean* distance between ionisations: $1 / \alpha$
    - All interactions playing their role
  - *Minimum* distance between ionisations: IP / E
    - Assuming only ionising collisions
  - mean ÷ minimum: $\kappa \overset{\text{def}}{=} E / \alpha.\mathrm{IP}$
- large κ no minimum distance effect → exponential,
- $\kappa \simeq 1$ no fluctuations → peaked.

Heinz Raether's group (Hamburg)

- After ionisation, electrons have to travel a minimum distance before their energy again suffices to ionise.

- $\kappa = E / \alpha.\mathrm{IP}$ is an indicator of the avalanche shape

- Lothar Frommhold (1956)

κ = 12-110:	exponential

- Hans Schlumbohm (1958)

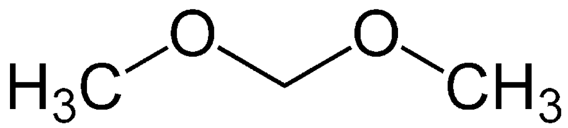
$\kappa > 23$:	exponential
$23 > \kappa > 10$:	levels off towards small sizes
$10 > \kappa$:	a maximum appears

- Werner Legler (1961)

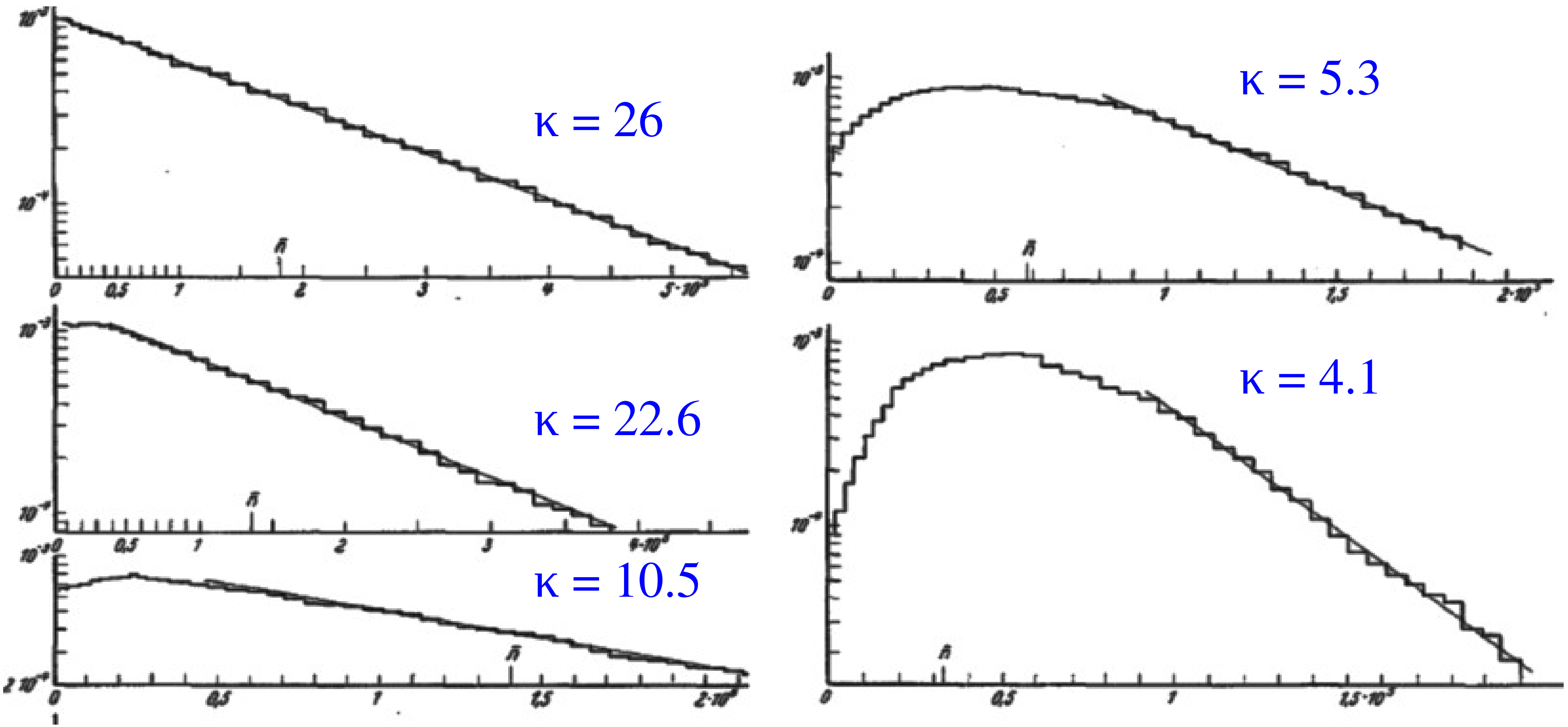
any κ	model calculation.

Heinz Artur Raether (1909-1986)

Hans Schlumbohm (1958)

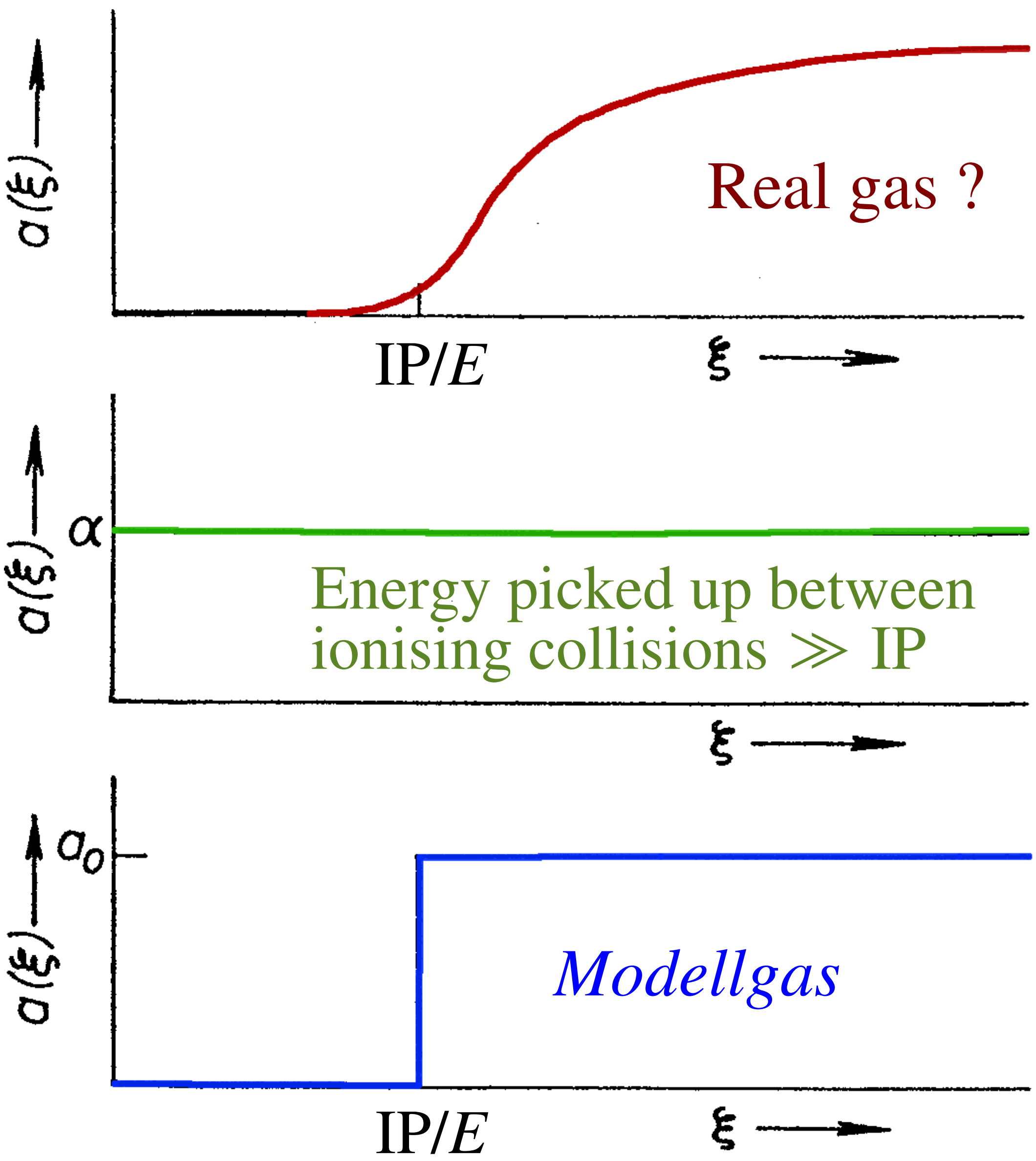


- Dimethoxymethane spectra: increasing *E*, decreasing *p d* and ~constant mean gain.



- Hans Schlumbohm, *Zur Statistik der Elektronenlawinen im ebenem Feld*, Z. Phys. **151** (1958) 563-576.

Werner Legler's *Modellgas* (1961)



ξ = distance since last ionisation;
$a(\xi)$ = probability to ionise again.

[Werner Legler, *Der Statistik der Elektronen-lawinen in electronegativen Gasen, bei hohen Feldstärken und bei großer Gasverstärkung,*

Z. Naturforschg. **16 a** (1961) 253-261.]

The *Magnettrommelrechner* (1961)

- Excellent agreement ... but no closed form

Die Rechnungen wurden mit dem Magnettrommelrechner IBM 650 des Instituts für Angewandte Mathematik der Universität Hamburg durchgeführt. Den an der Rechenanlage tätigen Mitarbeitern des Instituts danke ich für die Hilfe bei der Ausführung der Rechnung.

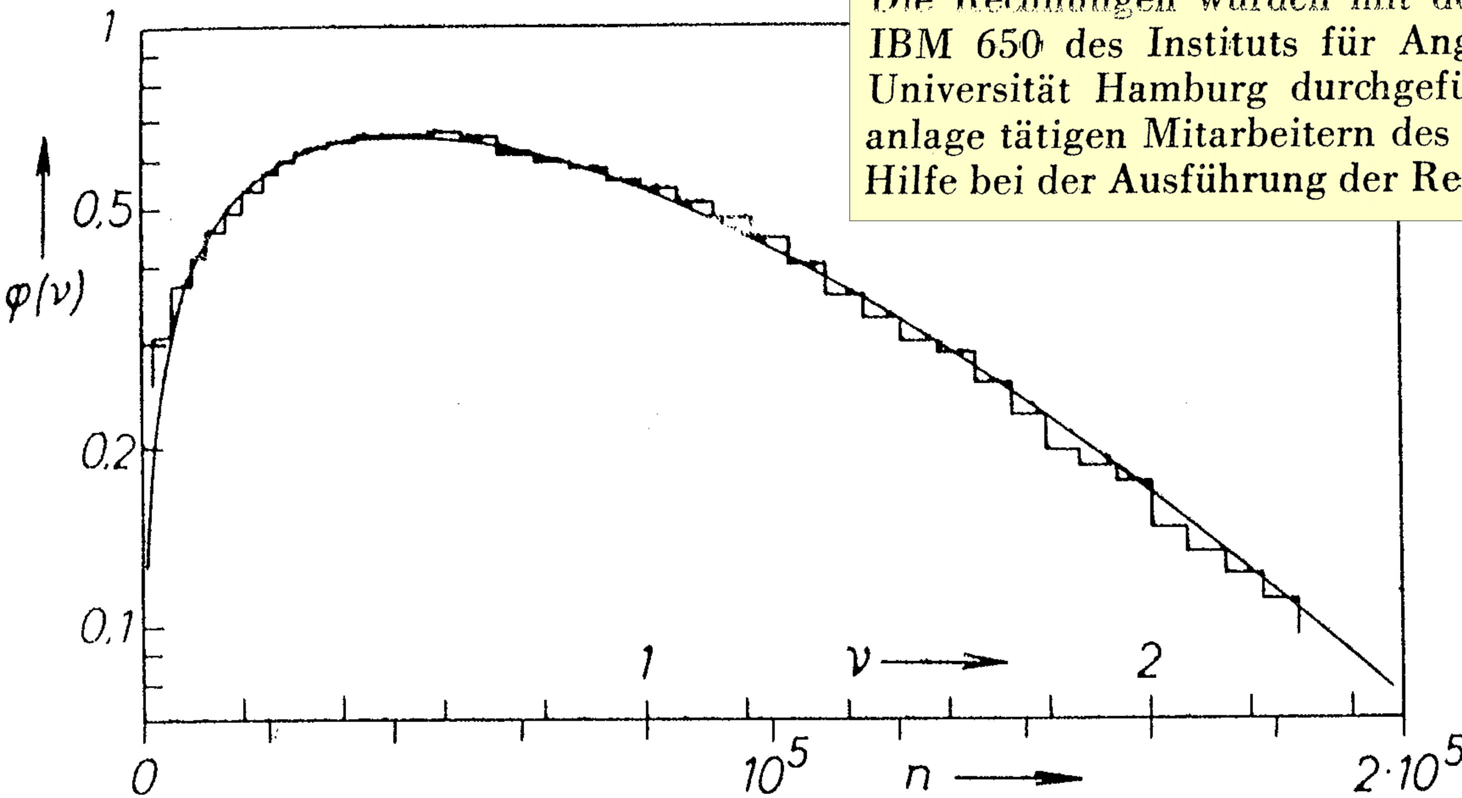


Abb. 5. Lawinenverteilung in Methylal nach SCHLUMBOHM [8]. $E/p = 186{,}5$ Volt/cm·Torr, $\alpha \cdot U_\mathrm{i}/E = 0{,}19$. Ausgezogene Kurve: Theoretische Verteilung im Modellgas für $\alpha\, x_0 = 0{,}18$.

$\kappa = 5.3$

The alternative school

- Townsend coefficient not constant ...

J. Byrne (1962)

- Observing that "the average energy of the two electrons coming from an ionizing collision must be less than the energy of the colliding electron", he chose the ansatz:

$$\alpha(r,n) = f(r)\left(a_0 + \frac{a_1}{n}\right)$$

- He then showed that for on-average-large avalanches, the Pólya distribution follows, which is in agreement with Curran's measurements.

- Note: J. Byrne published a different model in 1969.

A. Lansiart & J.P. Morucci (1962)

- Small avalanches are composed of electrons that
  - have ionised less, hence
  - have more energy, hence
  - will ionise more easily
- They modeled this with an avalanche size-dependent α:

$$\alpha(n) = \alpha(0)\left(1 + \frac{k}{n}\right)$$

- Implies that $(\sigma/\mu)^2 = 1/(1+k) < 1$, in agreement with Curran's measurements.
- Electron energy distribution continues to decrease, without reaching an equilibrium.

Werner Legler's response (1967)

“ To do this in general one has to use an ionization coefficient $\alpha(n, x)$ which depends not only on n but also on the distance x the avalanche has covered from the starting point (cathode) of the primary electron.

Besides the experimental doubts, the introduction instead of $\alpha(n, x)$ of an ionization coefficient which depends on n only leads to serious theoretical difficulties.

The suppression of the dependence on x means that the electron swarm has constant ionization probability between successive ionizations and relaxation effects are neglected, completely contrary to the intention of Cookson and Lewis.

Furthermore, a dependence of the ionization coefficient on n alone is understandable only if there are space-charge effects, and these are quite negligible at the beginning of the avalanche development.”

[W. Legler, *The influence of the relaxation of the electron energy distribution on the statistics of electron avalanches*, Brit. J. Appl. Phys. **18** (1967) 1275-1280,]

Г.Д. Алхазов (1970)

- "Statistics of electron avalanches and ultimate resolution of proportional counters", NIM **89** (1970) 155-165.

- Classic paper – examines various geometries, and the ionisation probability as function of distance traveled.

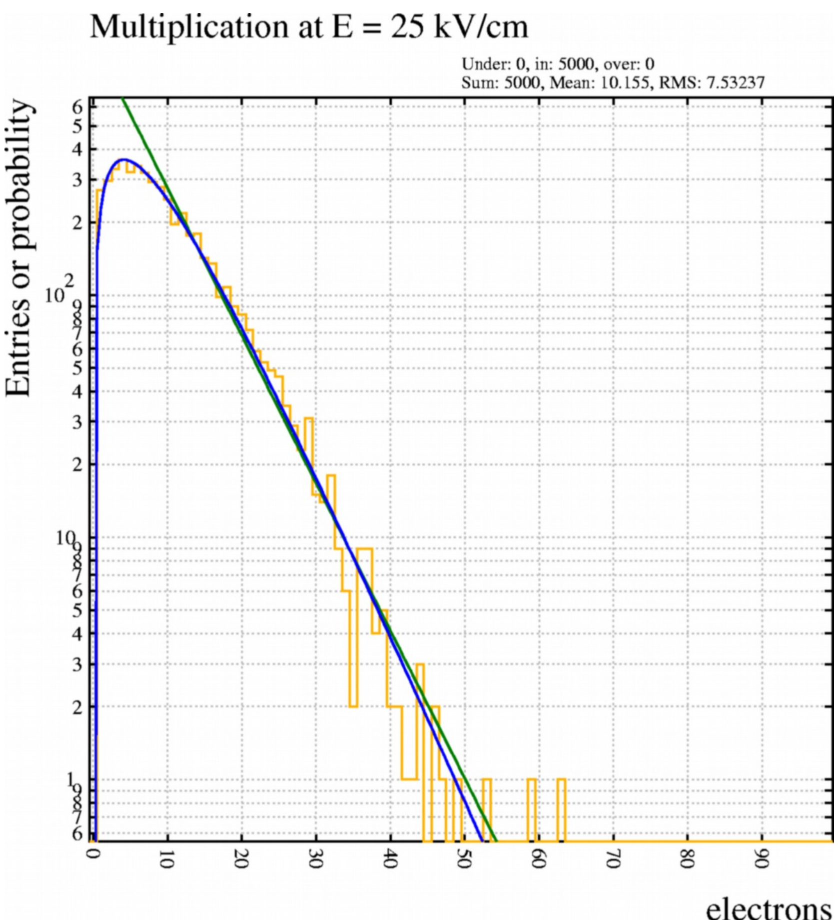
- [...] indeed there exists some correlation between α, and *K* [number of electrons already in the avalanche] but it has a much more complicated form as compared to that in eq. (3) [$\alpha \propto 1 + \mu/K$] so that the assumption that the ionization probability depends only on *K* is in principle unsuitable for the description of the electron avalanche statistics. [...] the distribution of the number of electrons in the single avalanche in uniform fields deviates from a Polya distribution. [...] In proportional cylindrical counters the distribution is in close agreement with a Polya one

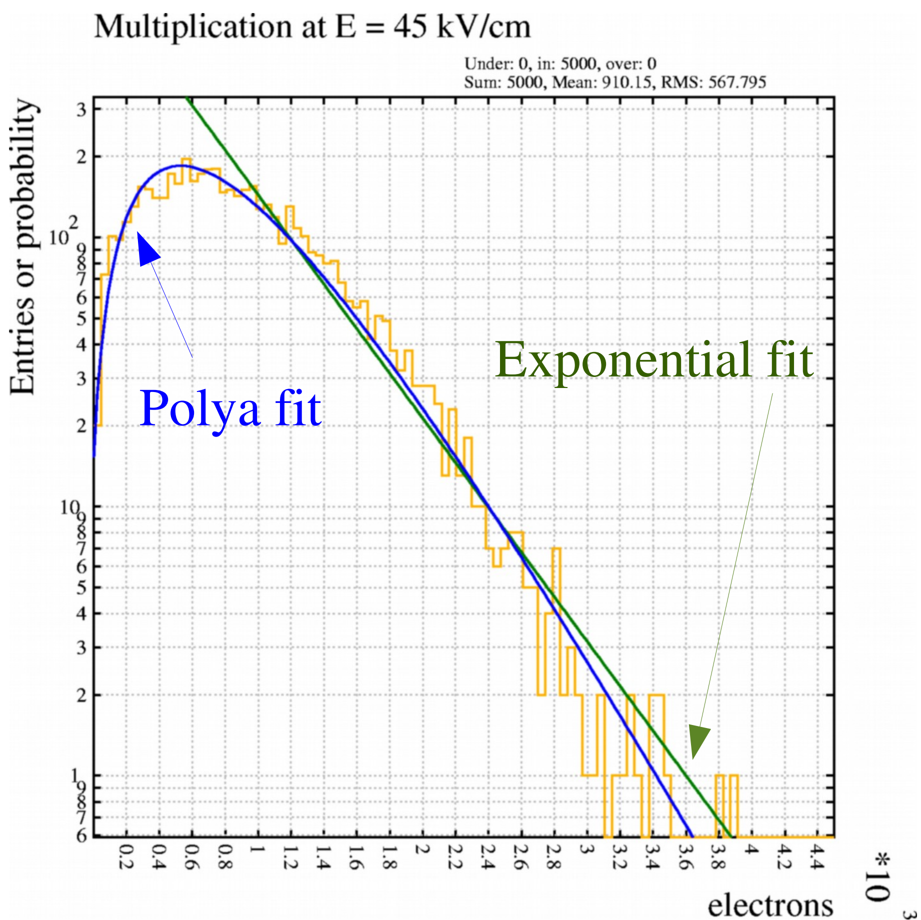
Monte Carlo approach – a way out ?

- Analytic models are precious for the insight they afford.
- But the complexity of real gases and detectors make realistic models unwieldy:
  - inelastic collisions (vibrations, rotations, polyads);
  - excitations and Penning transfers;
  - ionisation;
  - attachment;
  - intricate, position-dependent E and B fields.
- Predictions for experiments are more practical using a Monte Carlo approach, here based on Magboltz.

Pure argon: Magboltz distribution

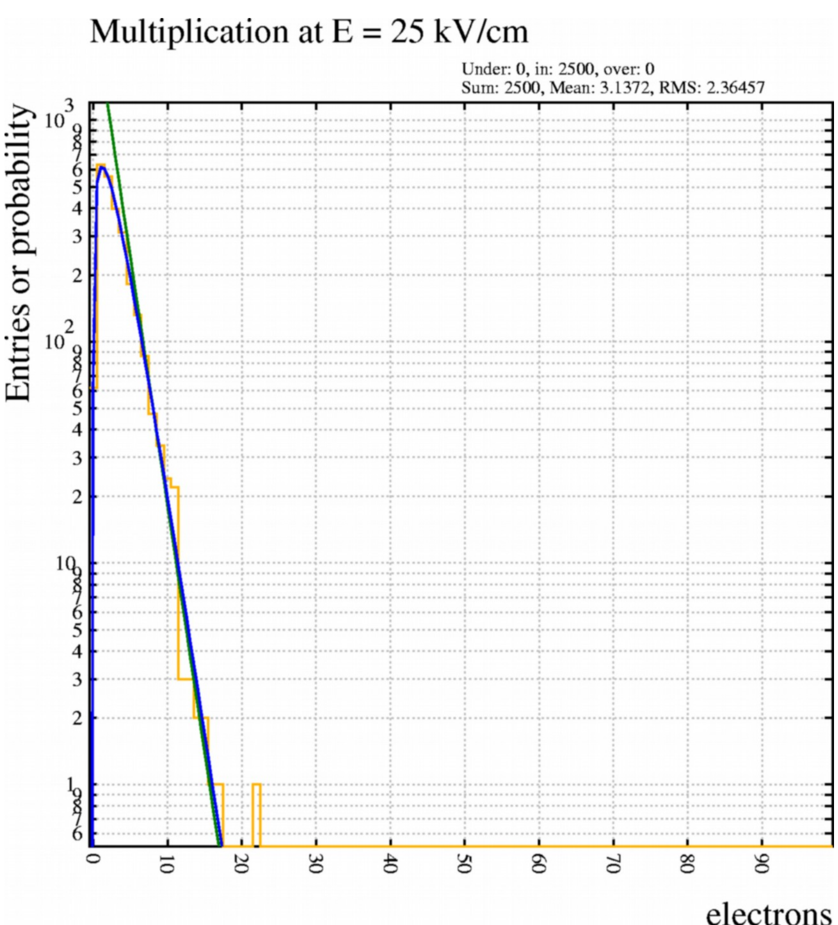
- With increasing E, $\kappa = E/\alpha.\mathrm{IP}$ decreases: the size distribution becomes more rounded (equal gap):

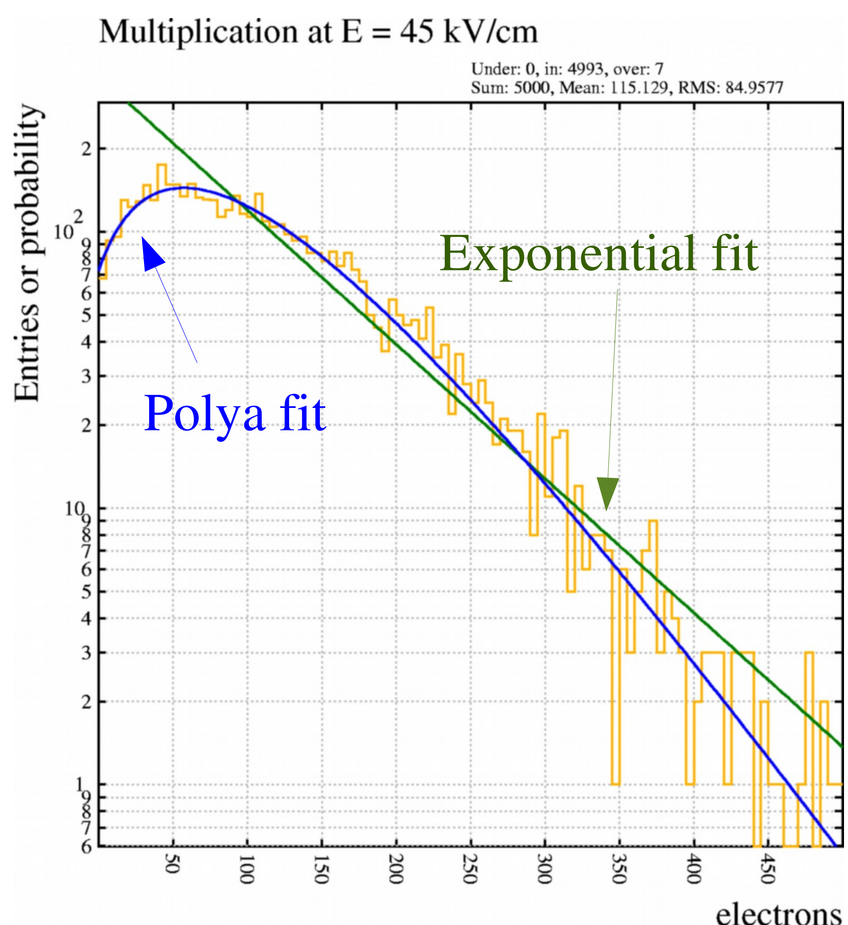




Ar/CO_2: size distribution

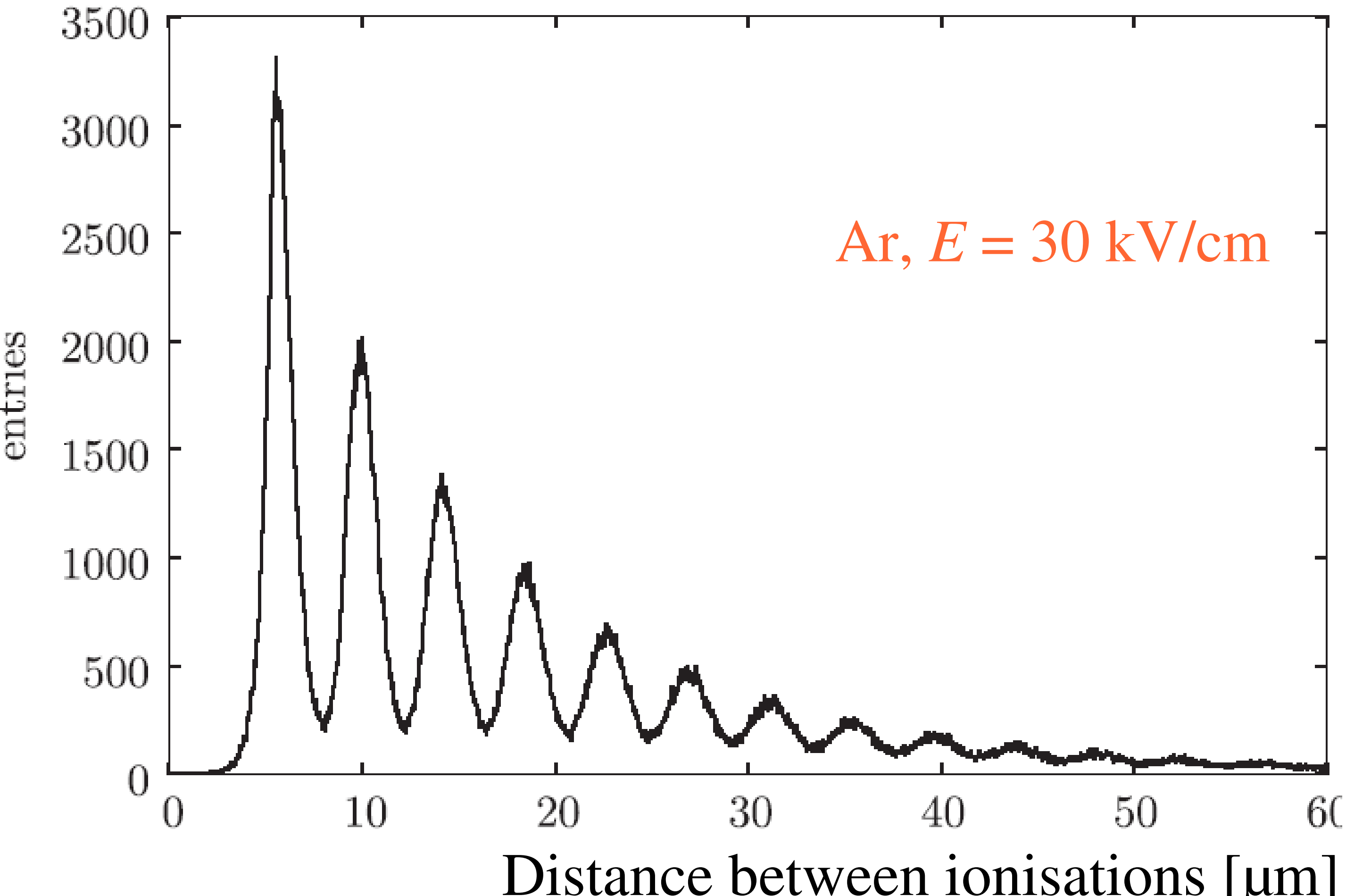
- Lower gain than pure Ar, but with increasing field, the size distribution still becomes more and more round:



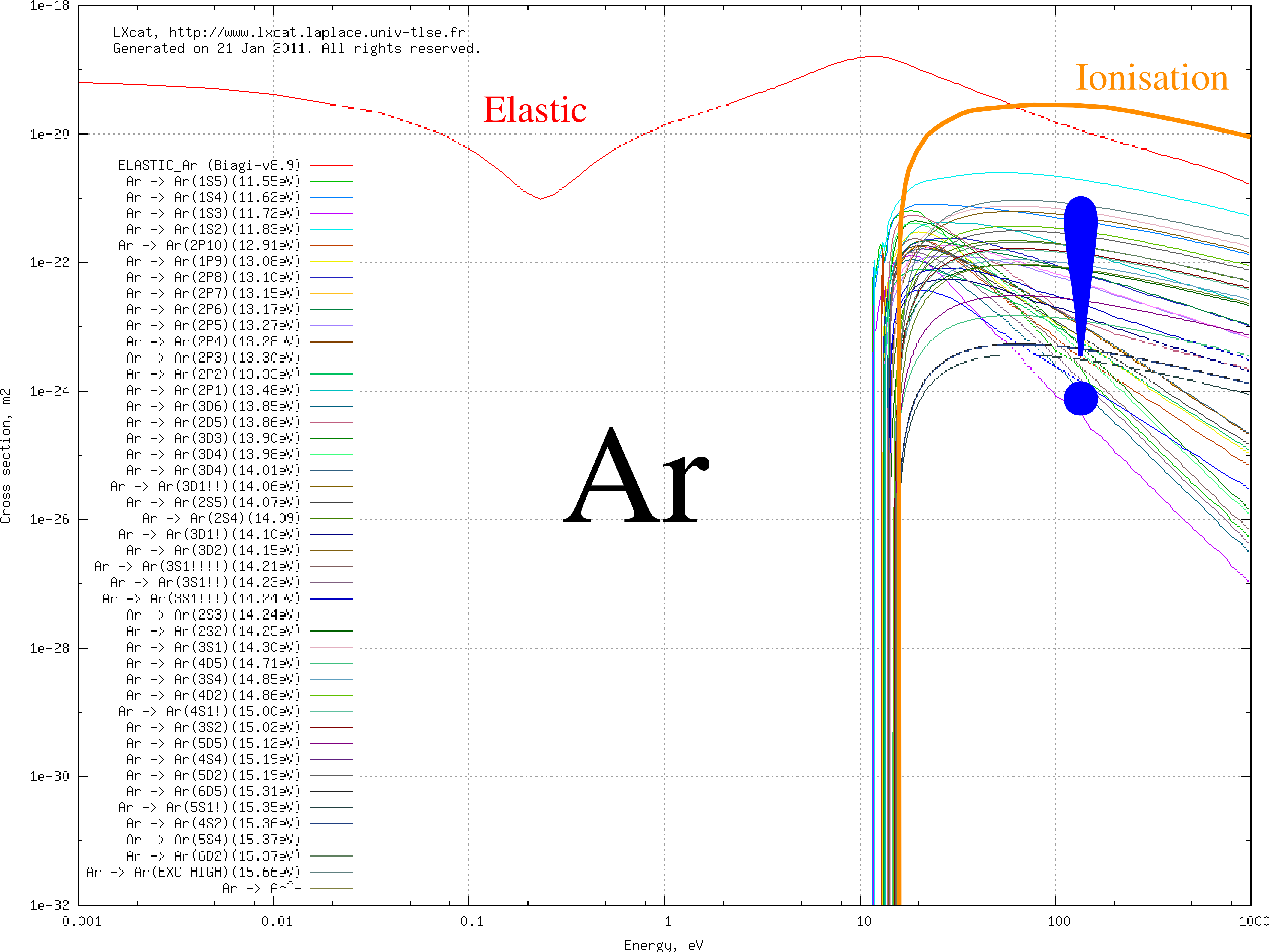


Distance between ionisation

- The distance between successive ionisations oscillates, shown here for Ar (also happens in CH_4 for instance).
- Why ?



[Magboltz calculations by Heinrich Schindler]



LXcat, http://www.lxcat.laplace.univ-tlse.fr
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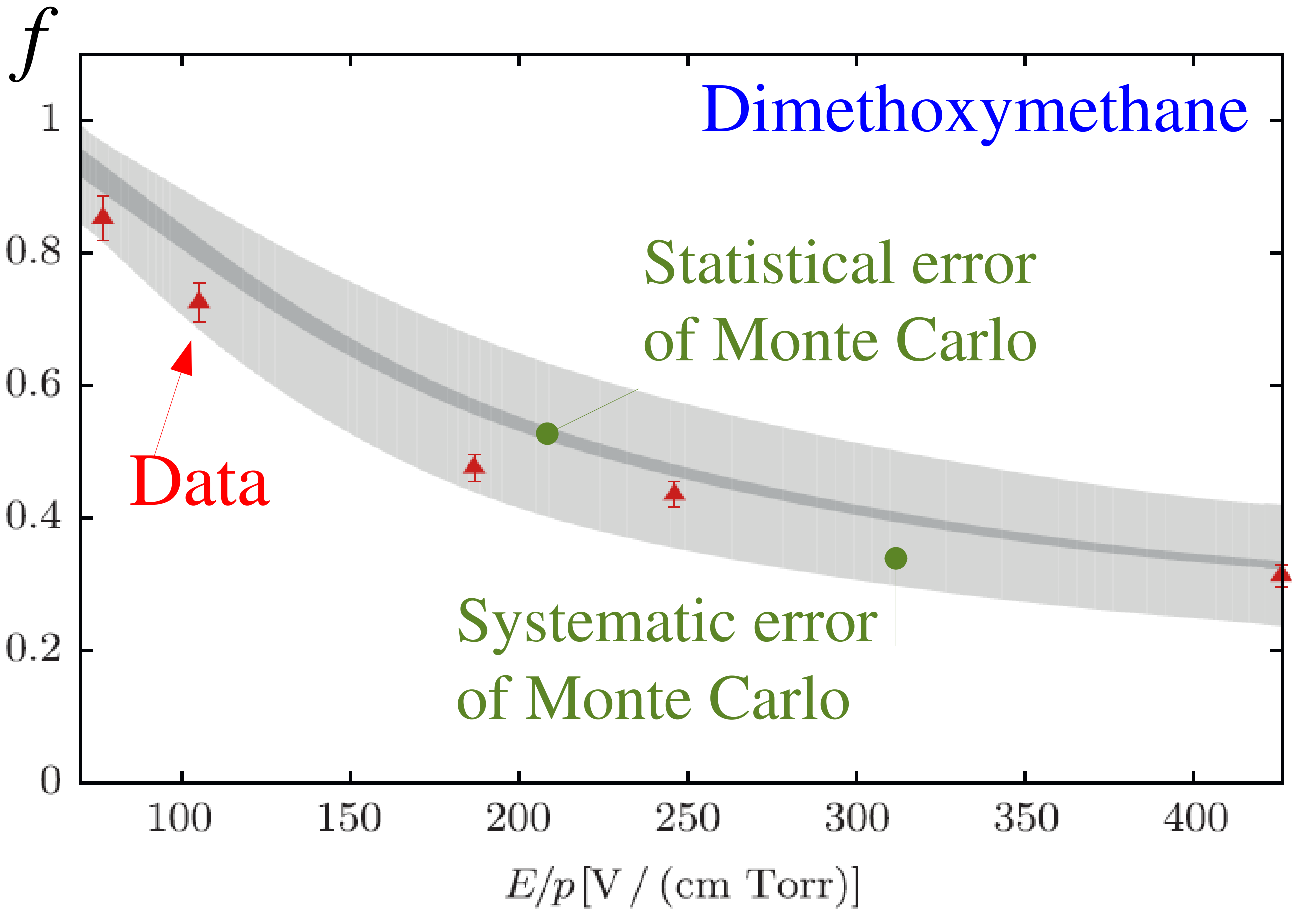
Elastic
Ionisation
Ar
!
ELASTIC_Ar (Biagi-v8.9)
Ar -> Ar(1S5)(11.55eV)
Ar -> Ar(1S4)(11.62eV)
Ar -> Ar(1S3)(11.72eV)
Ar -> Ar(1S2)(11.83eV)
Ar -> Ar(2P10)(12.91eV)
Ar -> Ar(1P9)(13.08eV)
Ar -> Ar(2P8)(13.10eV)
Ar -> Ar(2P7)(13.15eV)
Ar -> Ar(2P6)(13.17eV)
Ar -> Ar(2P5)(13.27eV)
Ar -> Ar(2P4)(13.28eV)
Ar -> Ar(2P3)(13.30eV)
Ar -> Ar(2P2)(13.33eV)
Ar -> Ar(2P1)(13.48eV)
Ar -> Ar(3D6)(13.85eV)
Ar -> Ar(2D5)(13.86eV)
Ar -> Ar(3D3)(13.90eV)
Ar -> Ar(3D4)(13.98eV)
Ar -> Ar(3D4)(14.01eV)
Ar -> Ar(3D1!!)(14.06eV)
Ar -> Ar(2S5)(14.07eV)
Ar -> Ar(2S4)(14.09)
Ar -> Ar(3D1!)(14.10eV)
Ar -> Ar(3D2)(14.15eV)
Ar -> Ar(3S1!!!!)(14.21eV)
Ar -> Ar(3S1!!)(14.23eV)
Ar -> Ar(3S1!!!)(14.24eV)
Ar -> Ar(2S3)(14.24eV)
Ar -> Ar(2S2)(14.25eV)
Ar -> Ar(3S1)(14.30eV)
Ar -> Ar(4D5)(14.71eV)
Ar -> Ar(3S4)(14.85eV)
Ar -> Ar(4D2)(14.86eV)
Ar -> Ar(4S1!)(15.00eV)
Ar -> Ar(3S2)(15.02eV)
Ar -> Ar(5D5)(15.12eV)
Ar -> Ar(4S4)(15.19eV)
Ar -> Ar(5D2)(15.19eV)
Ar -> Ar(6D5)(15.31eV)
Ar -> Ar(5S1!)(15.35eV)
Ar -> Ar(4S2)(15.36eV)
Ar -> Ar(5S4)(15.37eV)
Ar -> Ar(6D2)(15.37eV)
Ar -> Ar(EXC HIGH)(15.66eV)
Ar -> Ar^+
1e-18
1e-20
1e-22
1e-24
1e-26
1e-28
1e-30
1e-32
Cross section, m2
0.001
0.01
0.1
1
10
100
1000
Energy, eV

Relative variance $f \equiv \sigma^2/\bar{n}^2$

- f is the experimental measure of "roundness":

$f > 1$	attachment
$f = 1$	exponential
$f = 0$	no spread

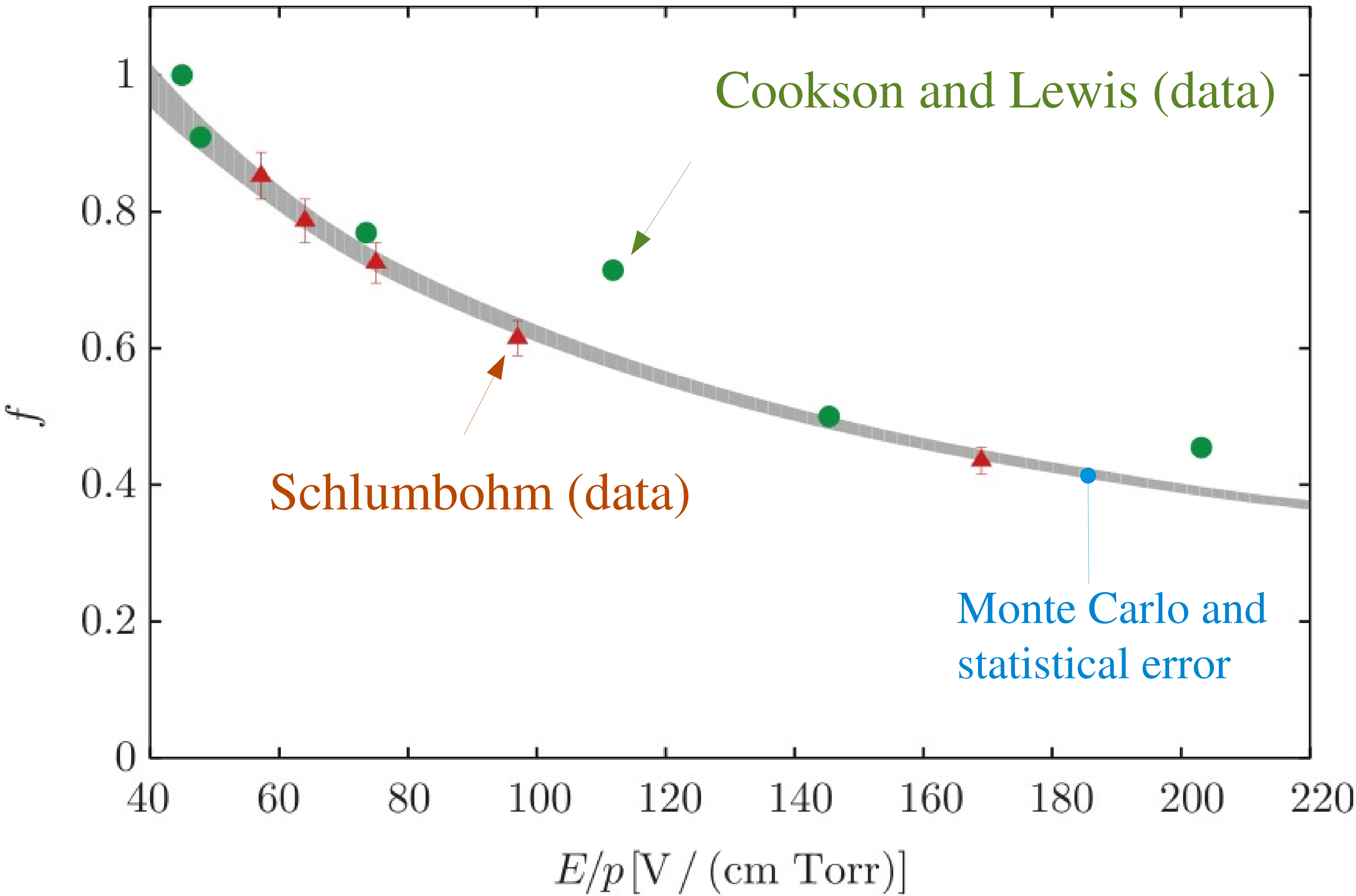


- $f \leftrightarrow \kappa$ translation:

$\kappa \gg 20 \quad f \approx 1$

$\kappa < 10 \quad f \downarrow 0$

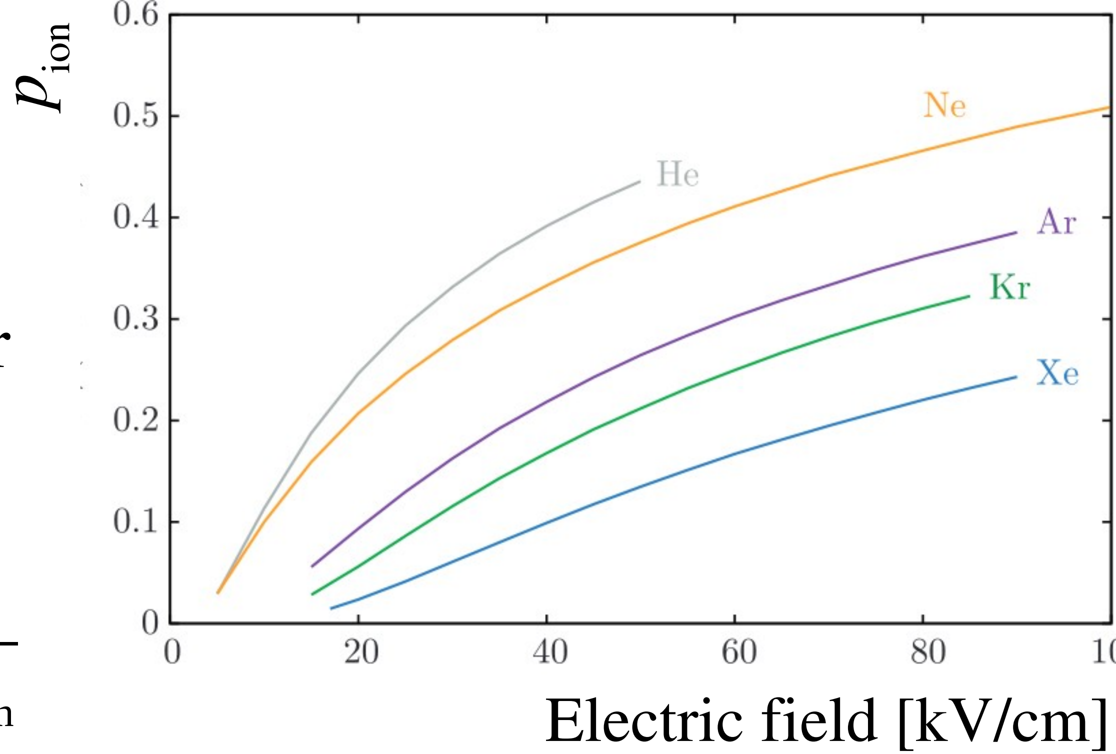
MC verification: methane

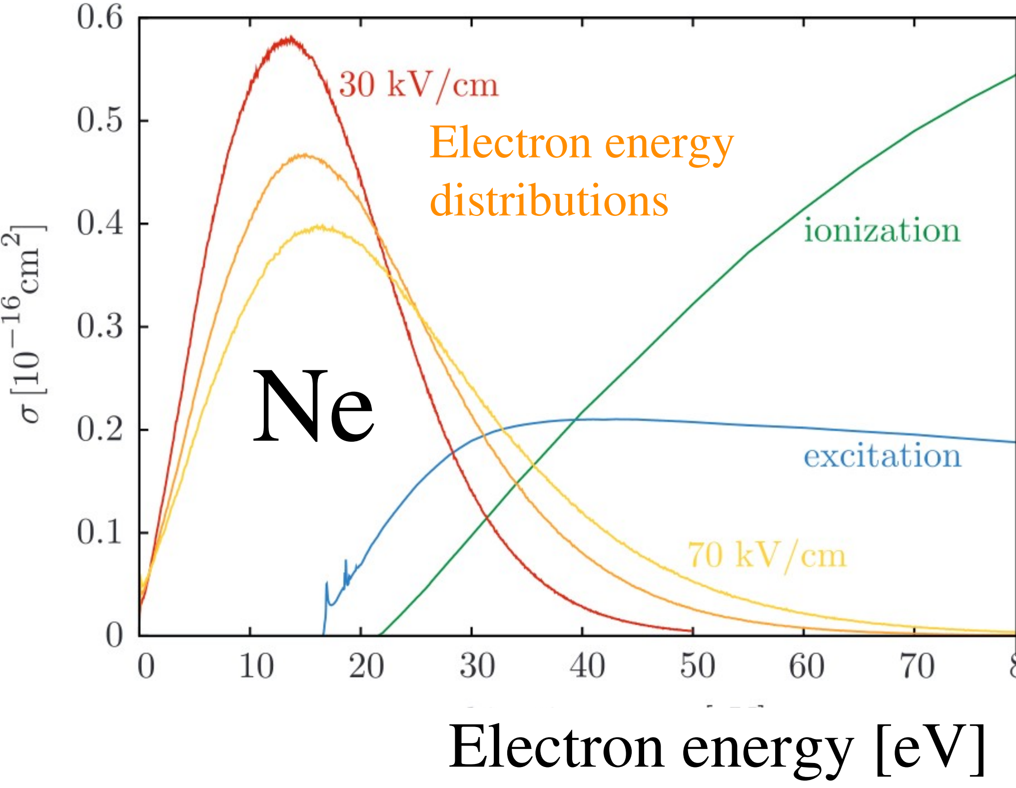


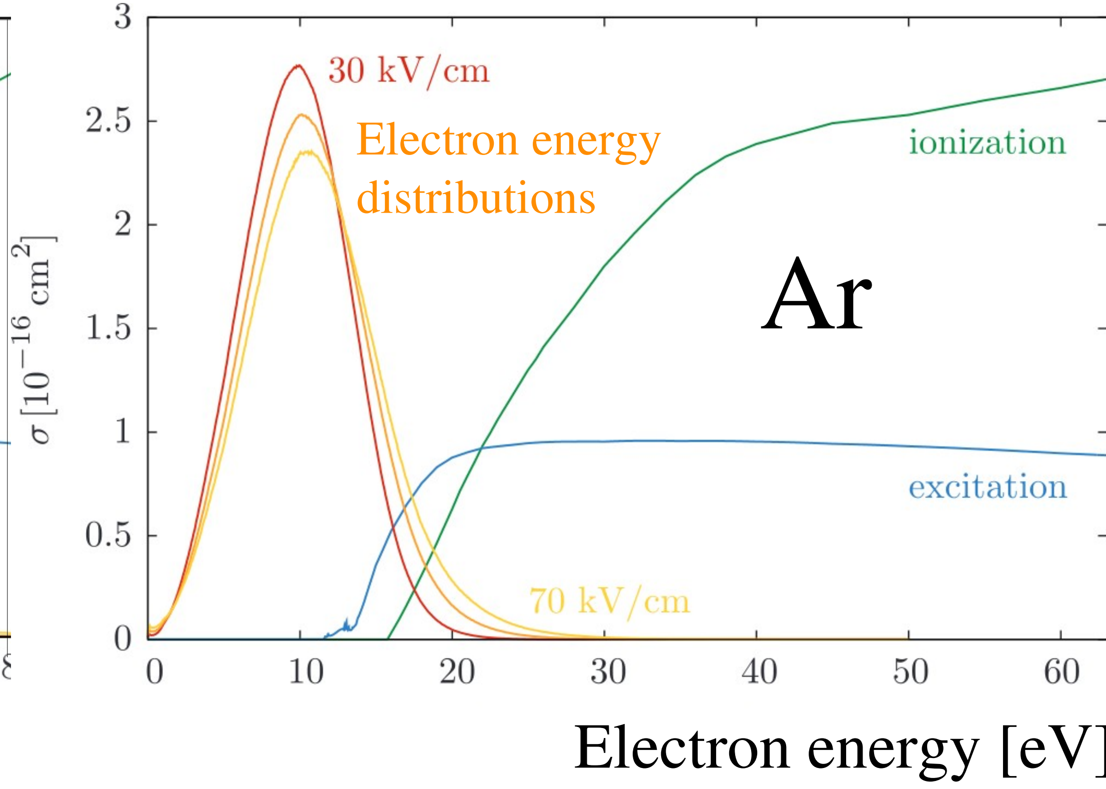
Noble gases

- Light gases are hot and favour ionisation. Hence f is lower.

$$f \approx \frac{1-p_{\text{ion}}}{1+p_{\text{ion}}} \quad \text{where} \quad p_{\text{ion}} \equiv \frac{\nu_{\text{ion}}}{\nu_{\text{exc}}+\nu_{\text{ion}}}$$

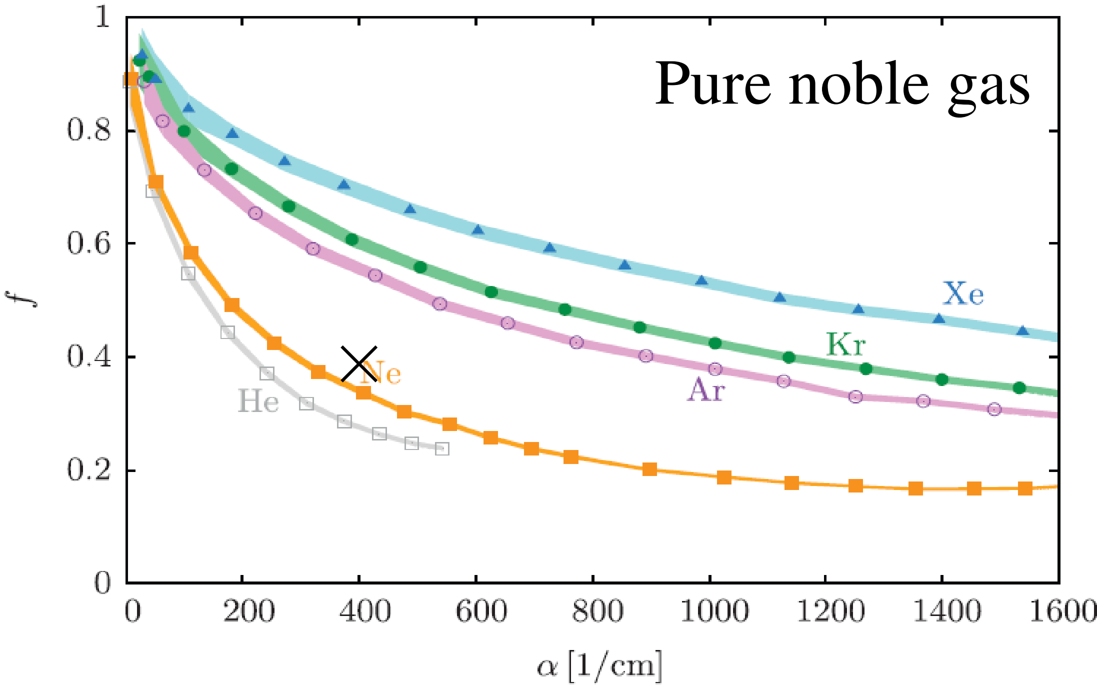


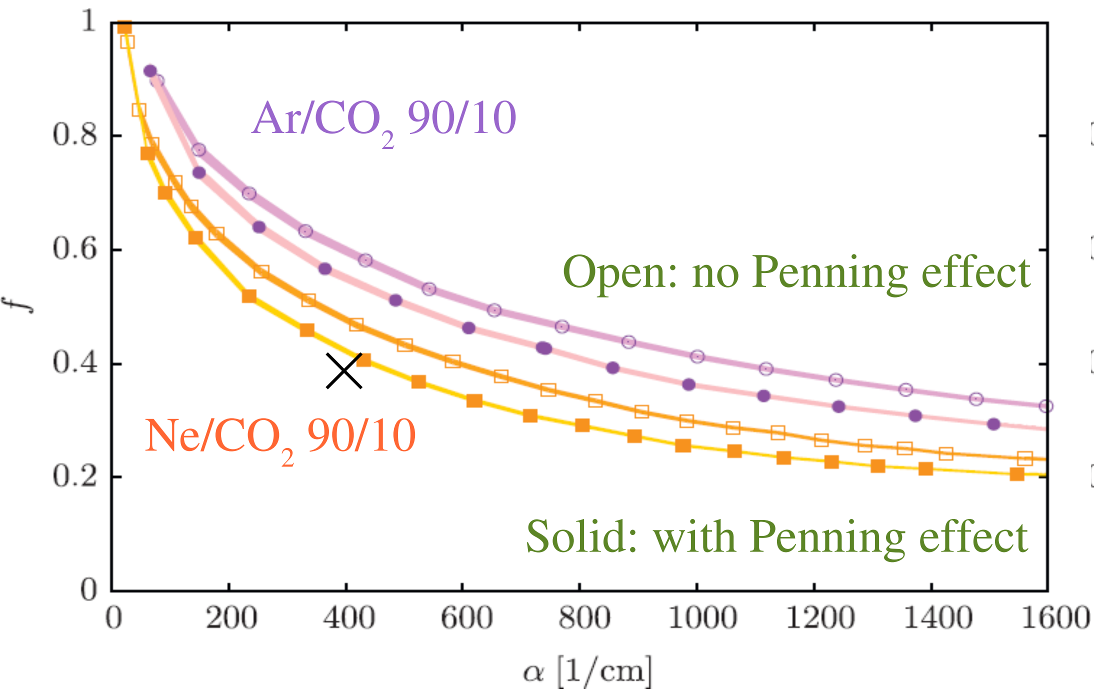


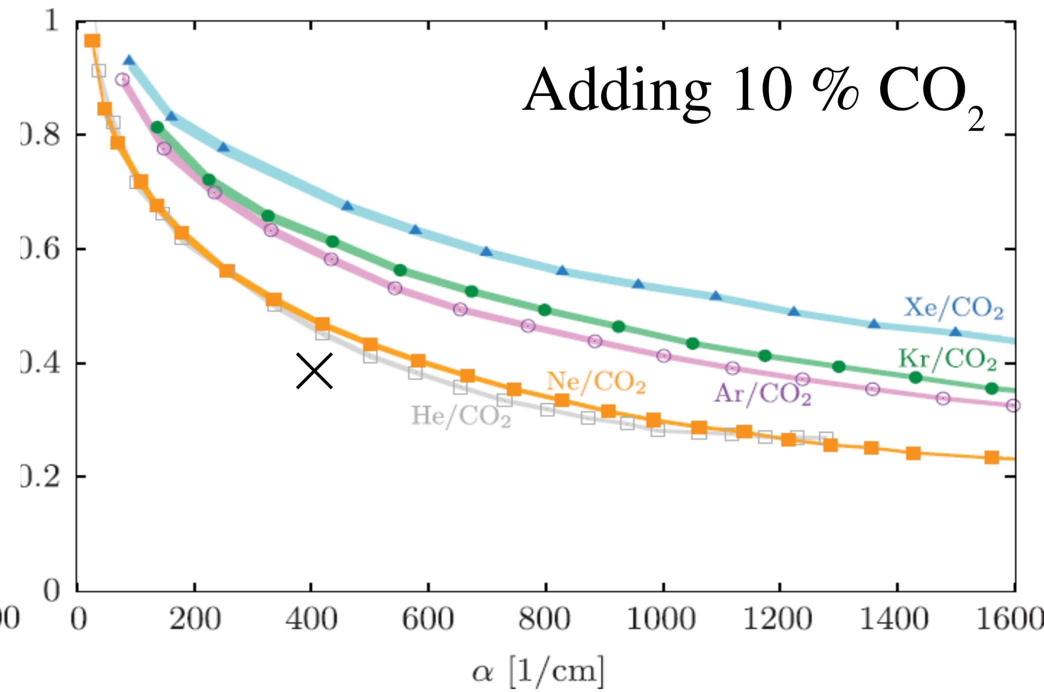


Effect of quenchers

- Quenchers: more inelastic & less ionisation → larger f ;
- Penning transforms excitation into ionisation → smaller f.







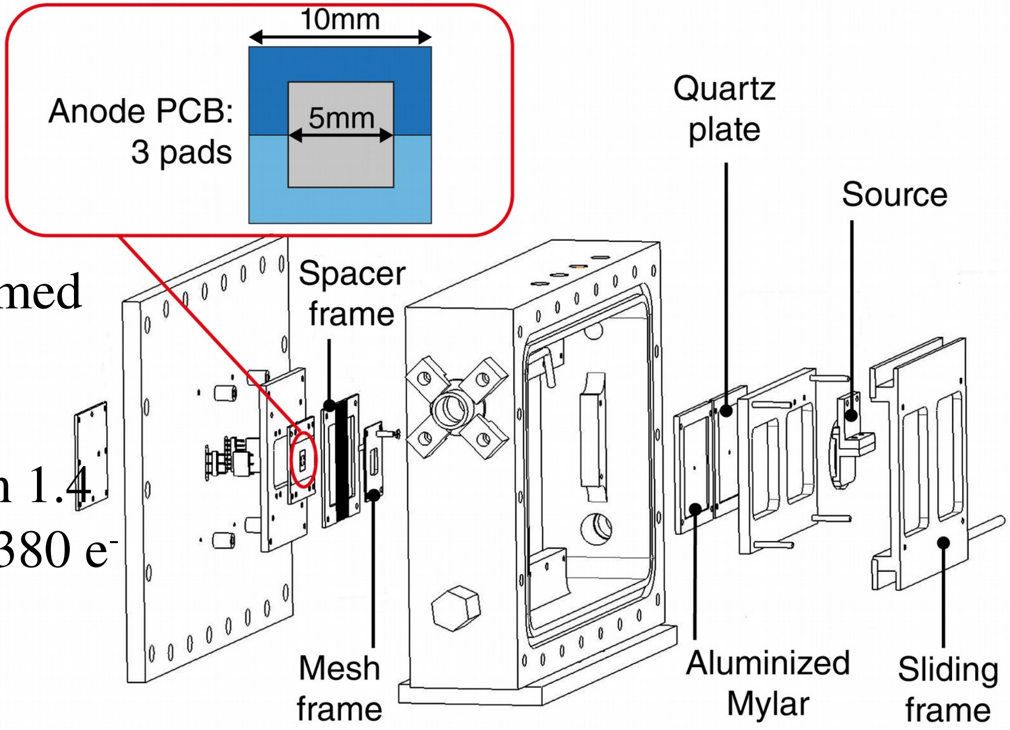
Factors that dis/favour a hump

- Exponential ($f \approx 1$) when electrons travel longer between ionisations than needed to acquire ionisation energy ($\kappa \gg 20$):
  - energy loss in the form of excitations;
  - heavy noble gases (excitation favoured over ionisation);
  - quenched gases: lower electron energy hence more excitation and less ionisation.
- Prominent hump ($f \downarrow 0$) when ionisation is prompt ($\kappa < 10$):
  - high electric field (more ionisation than excitation);
  - light noble gases (excitation is less favoured);
  - less quencher (higher electron energy);
  - efficient recovery of excitation energy (Penning).

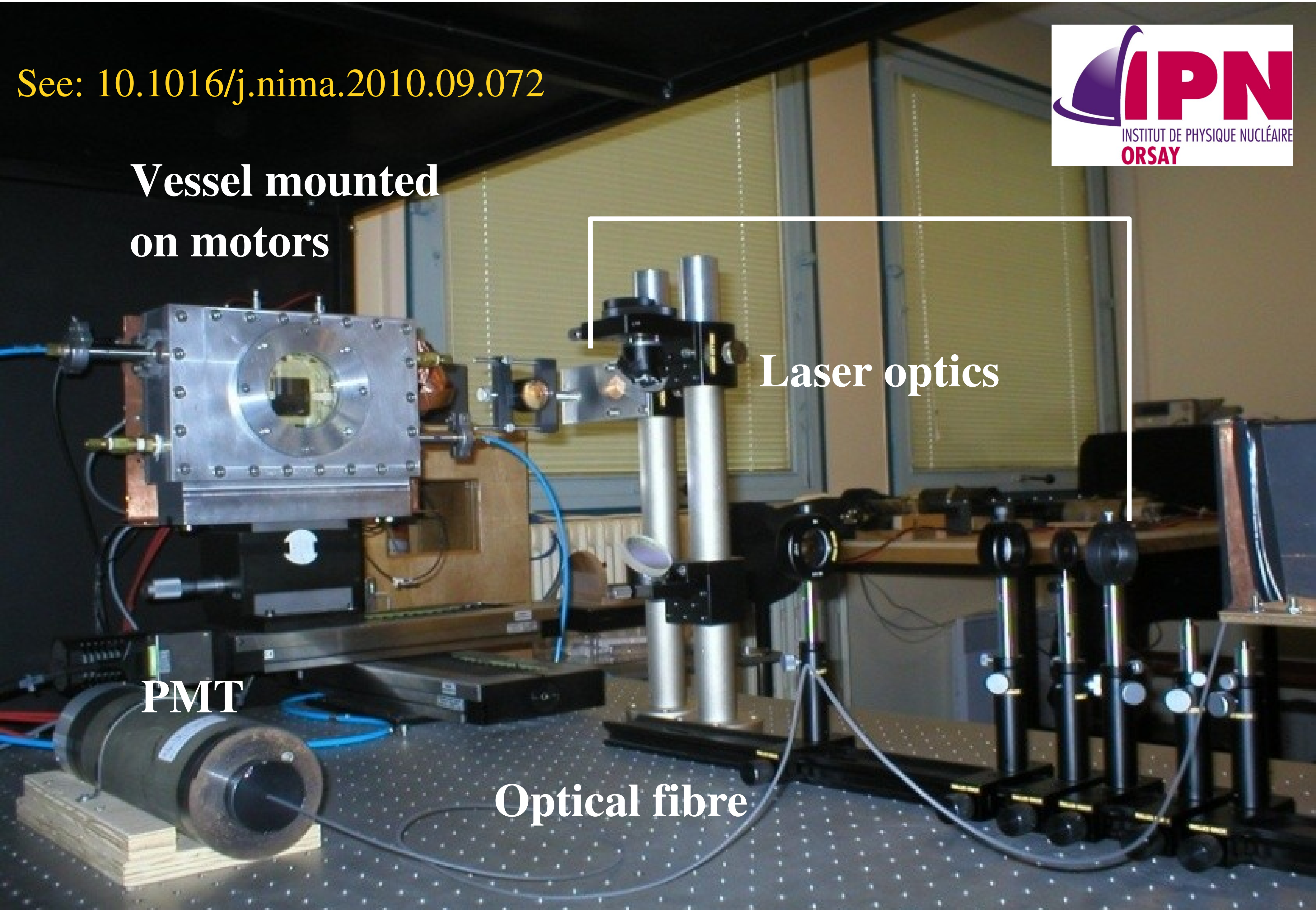
[See: 10.1016/j.nima.2010.09.072]

Measurement equipment

- Laser:
  - wave length: 337 nm (3.7 eV, i.e. well below the work functions of Ni and Cr: relies on two-photon interaction);
  - intensity lowered to ensure events with 2 electrons are exceedingly rare;
  - spot < 100 µm, duration: 4 ns FWHM.
- Gaps:
  - window: quartz + 0.5 nm NiCr;
  - drift: 3.2 mm;
  - amplification: 160 µm.
- Mesh:
  - Buckbee Mears 333 lpi electro-formed Ni Micro-Mesh™.
- Electronics:
  - pre-amplifier: Cremat CR-110 with 1.4 V/pC gain and 200 e^- RMS noise (380 e^- when hooked up);
  - amplifier: CAEN N568B.



Experimental setup



Single-electron spectra

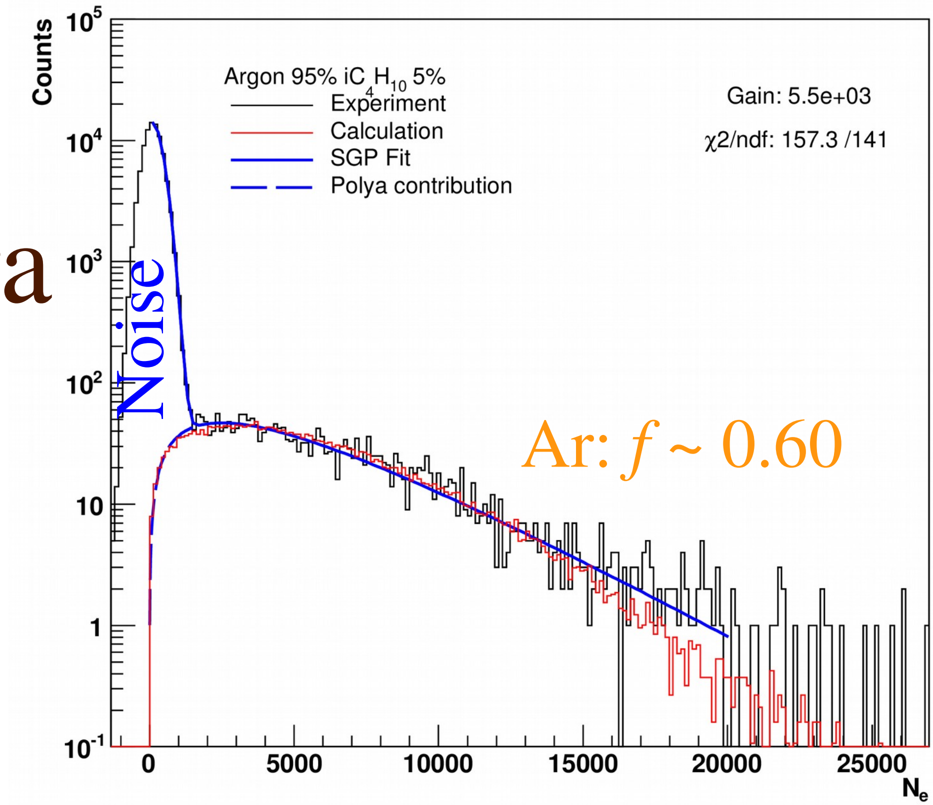
blue: Pólya signal + Gaussian noise fit;

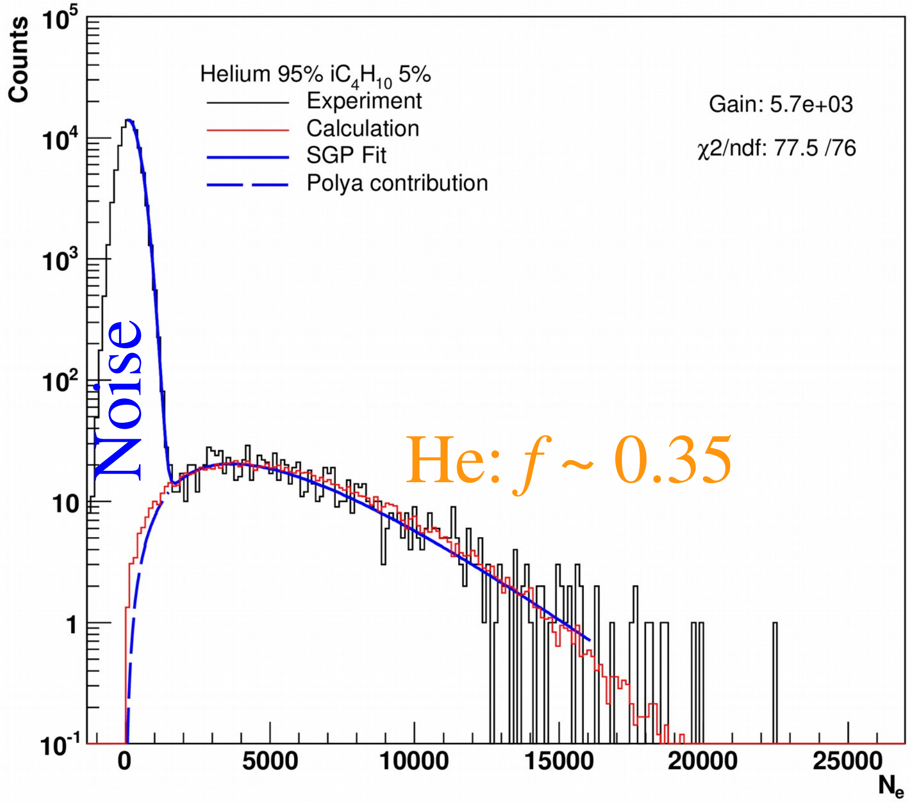
red: Monte Carlo (Magboltz), not fits !

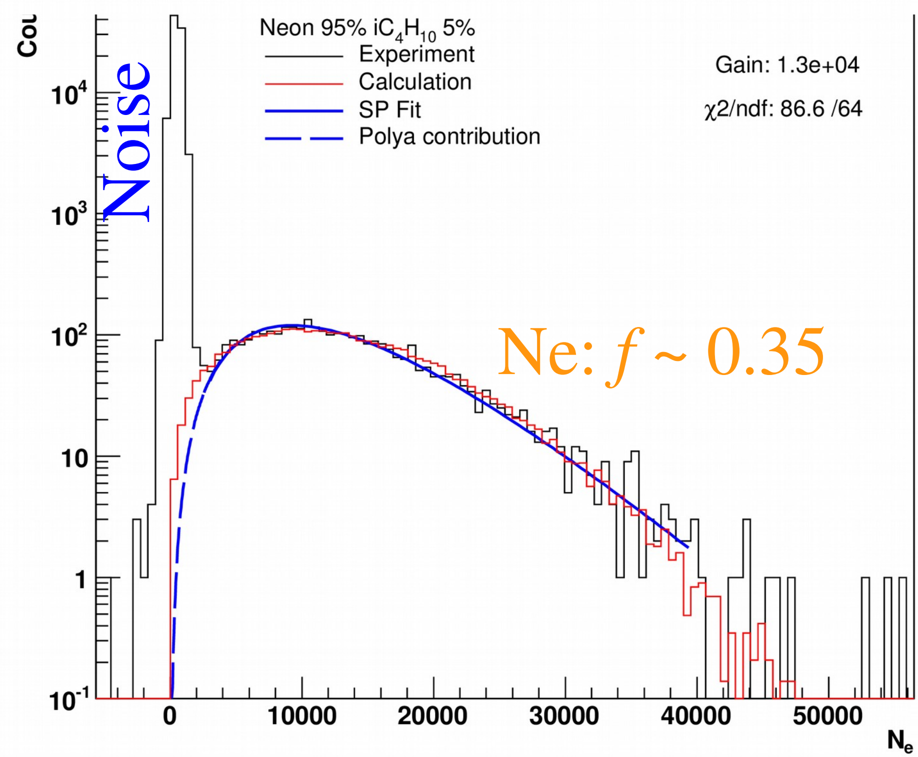
Ar 95 % $i\mathrm{C_4H_{10}}$ 5 %, E=28.12 kV/cm,

Ne 95 % $i\mathrm{C_4H_{10}}$ 5 %, E=26.25 kV/cm,

He 95 % $i\mathrm{C_4H_{10}}$ 5 %, E=26.25 kV/cm,



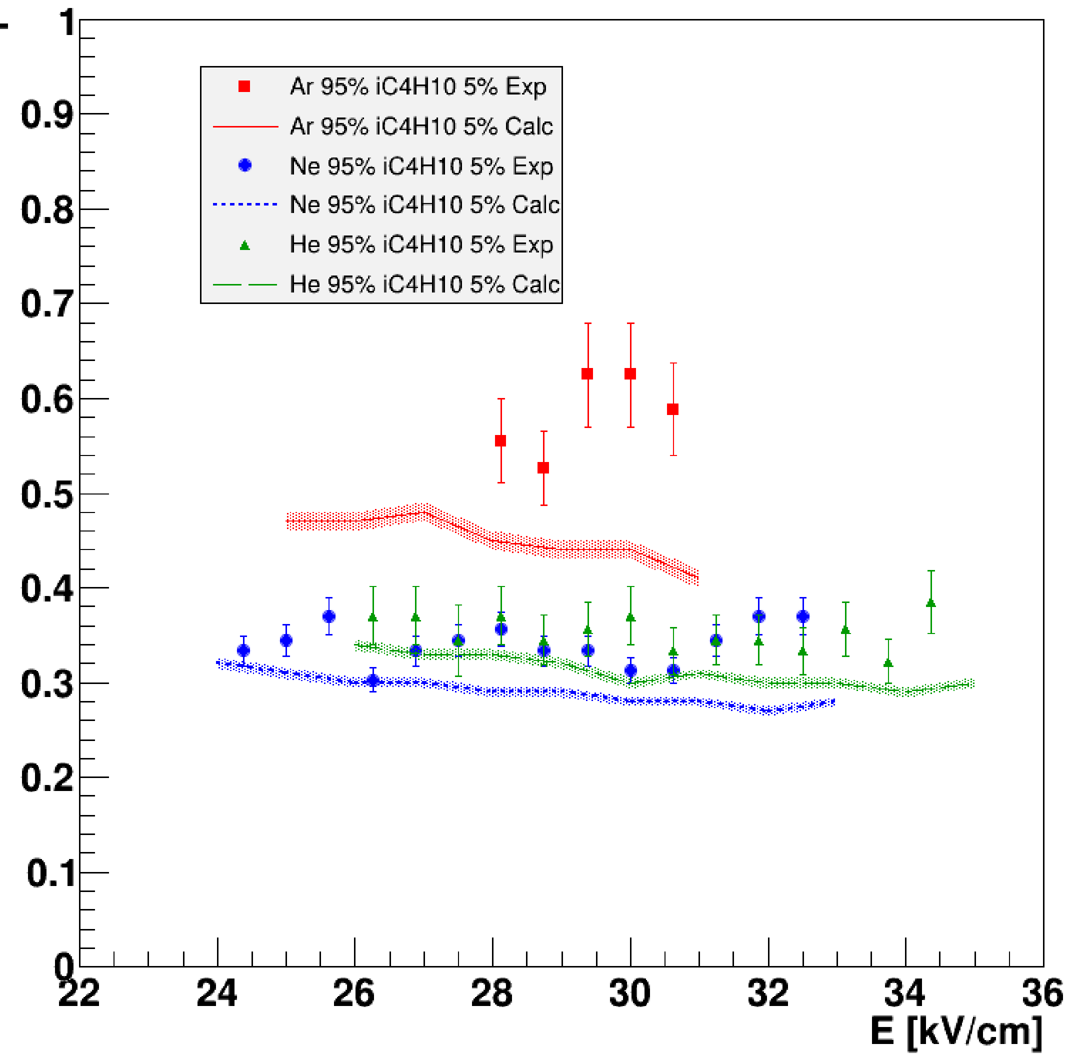




Relative variance *f*

- Ne and He more peaked than Ar, as expected from calculations.

- Measured and calculated relative variance *f* agree, except for Ar, in part due to the onset of discharges.



Summary

- A microscopic Monte Carlo reproduces several features. The moments of the full avalanche size distribution can only be extrapolated from smaller avalanches if energy relaxation is not an issue.

- The hump is more pronounced in the Ar mixture than in the He and Ne mixtures because
  - heavier gases have a lower ionisation yield;
  - the large Ar excitation losses are only in part recovered;
  - $i\mathrm{C_4H_{10}}$ neutral dissociation losses are larger with Ar.