Modelling of dynamic and transient behaviours of gaseous detectors

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Overview

Brief introduction
Definition of a general model
Description of the tools used
Some application examples
Discussion

Modelling detectors

Several reasons. In general for optimisation. Interests might be:
- gas gain
- detection efficiency
- signal generation, propagation and integrity
- position resolution
- time resolution
- energy resolution
- performance stability
- ageing effects
- maximum gain
- high particle flux capabilities
- …

Let me start from far

Macroscopic convection and diffusion of gases

Impractical to consider the evolution of the position and the momentum of each molecule interacting with the others

Quantities like pressure, density and temperature completely define the macroscopic behaviour of the gas

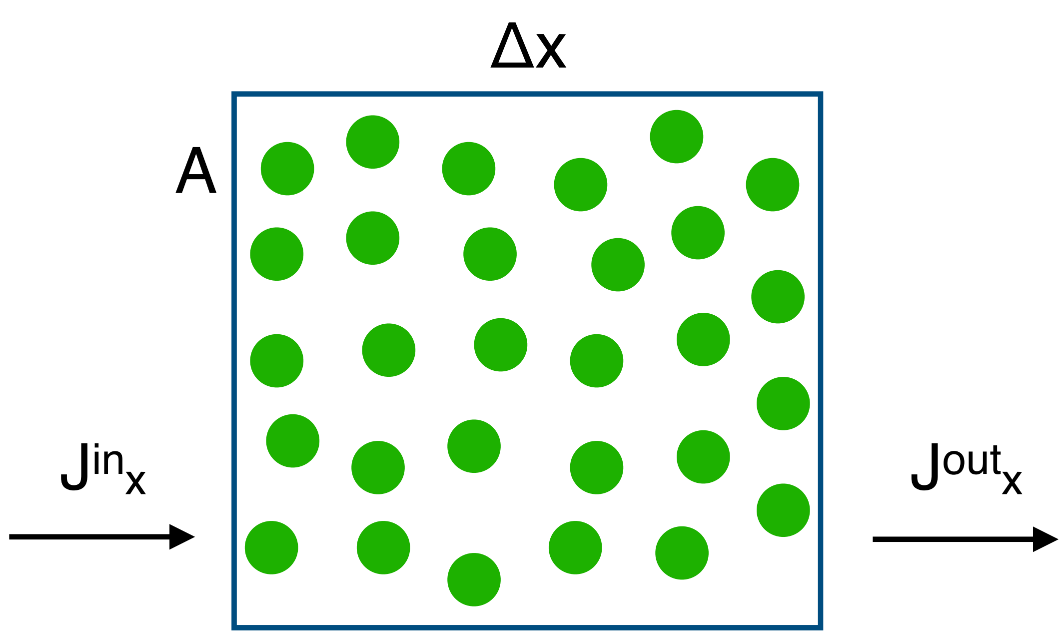
Number of molecules in the **typical volume of interest** is large
Mean free path is shorter than the **typical feature size**
Gas can be treated as a **continuum**

Let’s not get confused

The purpose is to model **gaseous detectors**, not gases

Continuity equation

Link the time derivative with spatial derivative



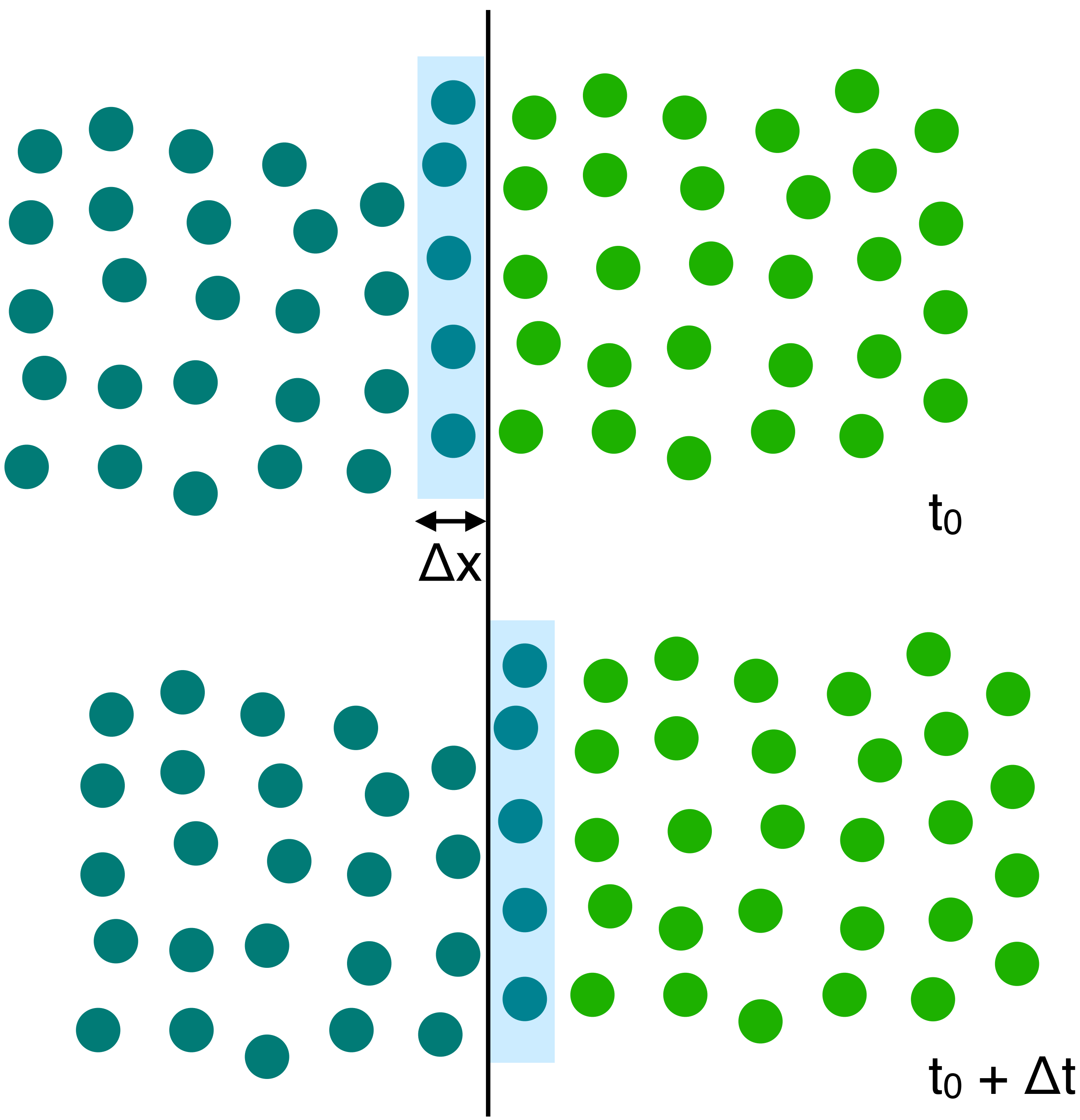
$$\frac{\Delta N}{\Delta t} = \frac{N_x^{in} - N_x^{out}}{\Delta t} = (J_x^{in} - J_x^{out})A$$

$$\frac{\Delta N}{\Delta t \cdot A \cdot \Delta x} = \frac{\Delta \rho}{\Delta t} = \frac{J_x^{in} - J_x^{out}}{\Delta x}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial J_x}{\partial x}$$

$$\frac{\partial \rho}{\partial t} = -(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}) = -\vec{\nabla} \cdot \vec{J}$$

Convection flux

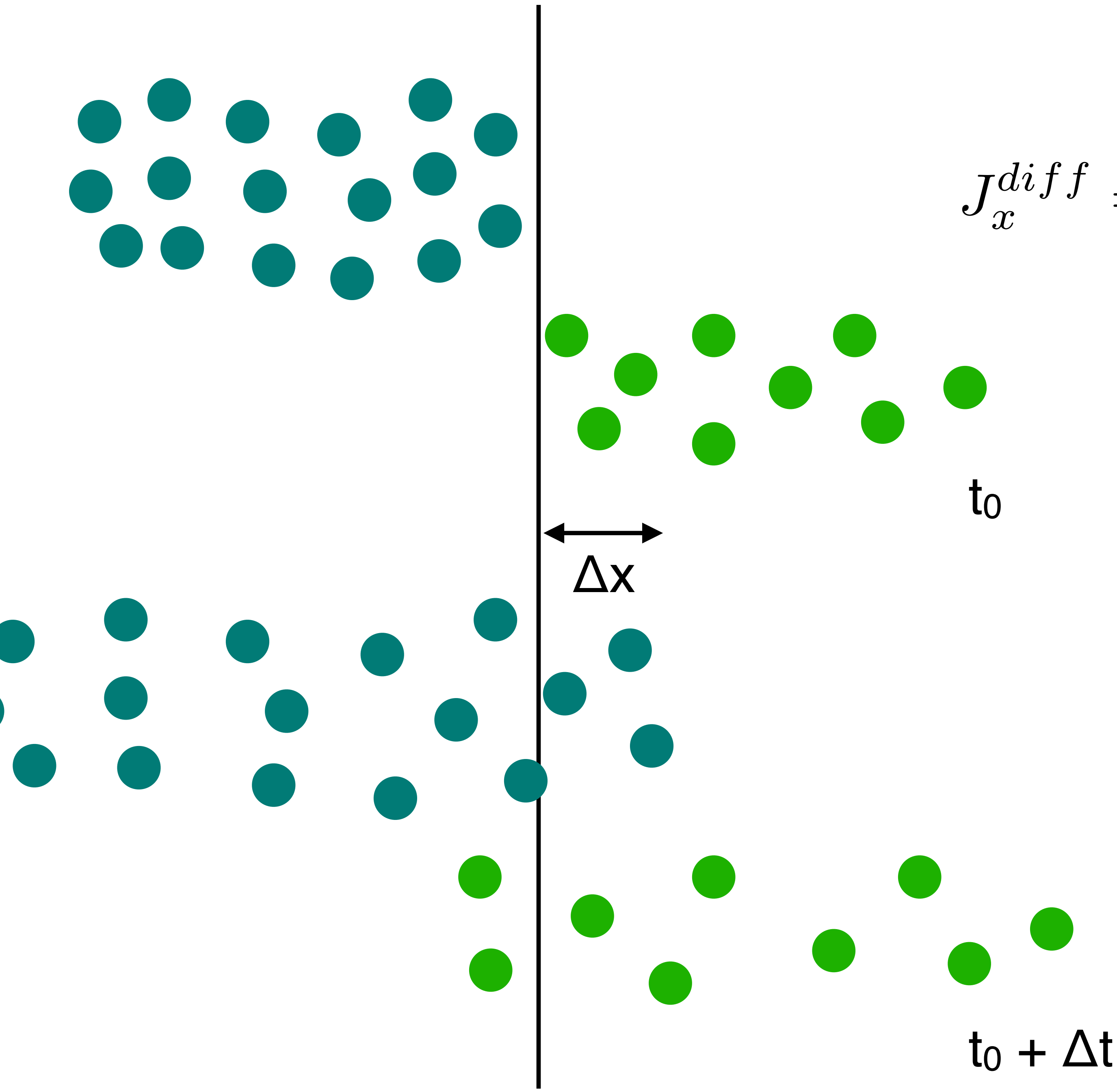


$$\frac{N_x^{in}}{\Delta t} = \rho A \frac{\Delta x}{\Delta t}$$

$$\frac{N_x^{in}}{A\Delta t} = J_x^{conv} = \rho v_x$$

$$\vec{J}^{conv} = \rho \vec{v}$$

Diffusive flux



$$J_x^{diff} = \frac{\Delta N_x}{A \Delta t} = (\rho_1 - \rho_2)\tilde{v}_x \delta = -\frac{\Delta \rho}{\Delta x} \Delta x \tilde{v}_x \delta$$

$$D_x := \Delta x \tilde{v}_x \delta$$

$$J_x^{diff} = -D_x \frac{\partial \rho}{\partial x}$$

$$\vec{J}^{diff} = -D \vec{\nabla} \rho$$

Sources, sinks and other

$$\frac{\partial \rho}{\partial t} = R$$

$$R = R(\vec{x}, t)$$

Sources or sinks

$$R = K(\vec{x}, t)\rho$$

“decay”

Everything together

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v} - D \vec{\nabla} \rho) + R$$

ρ is the unknown

For what concerns the boundary conditions, some examples

$\vec{n} \cdot \vec{J}^{tot} = 0$	No flux
$-\vec{n} \cdot \vec{J}^{tot} = \Phi(\vec{x}, t)$	Input flux
$\vec{n} \cdot \vec{J}^{diff} = 0$	Output
$\rho = \Psi(\vec{x}, t)$	Concentration

Plus symmetry and periodic conditions

The point is

Can the electrons and ions in the gas be considered as a gas?

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Unfortunately no

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Can the electrons and ions in the gas be considered as a gas?

Unfortunately no

But there are situation where it is a useful approximation

Why not

Electrons and ions cannot be considered continuous
- Number of electron and ion pairs not large
- Mean free path not necessarily small

Microscopic processes are stochastic
Solution of differential equation is deterministic

Why yes

When interested in an ‘**average**’ behaviour

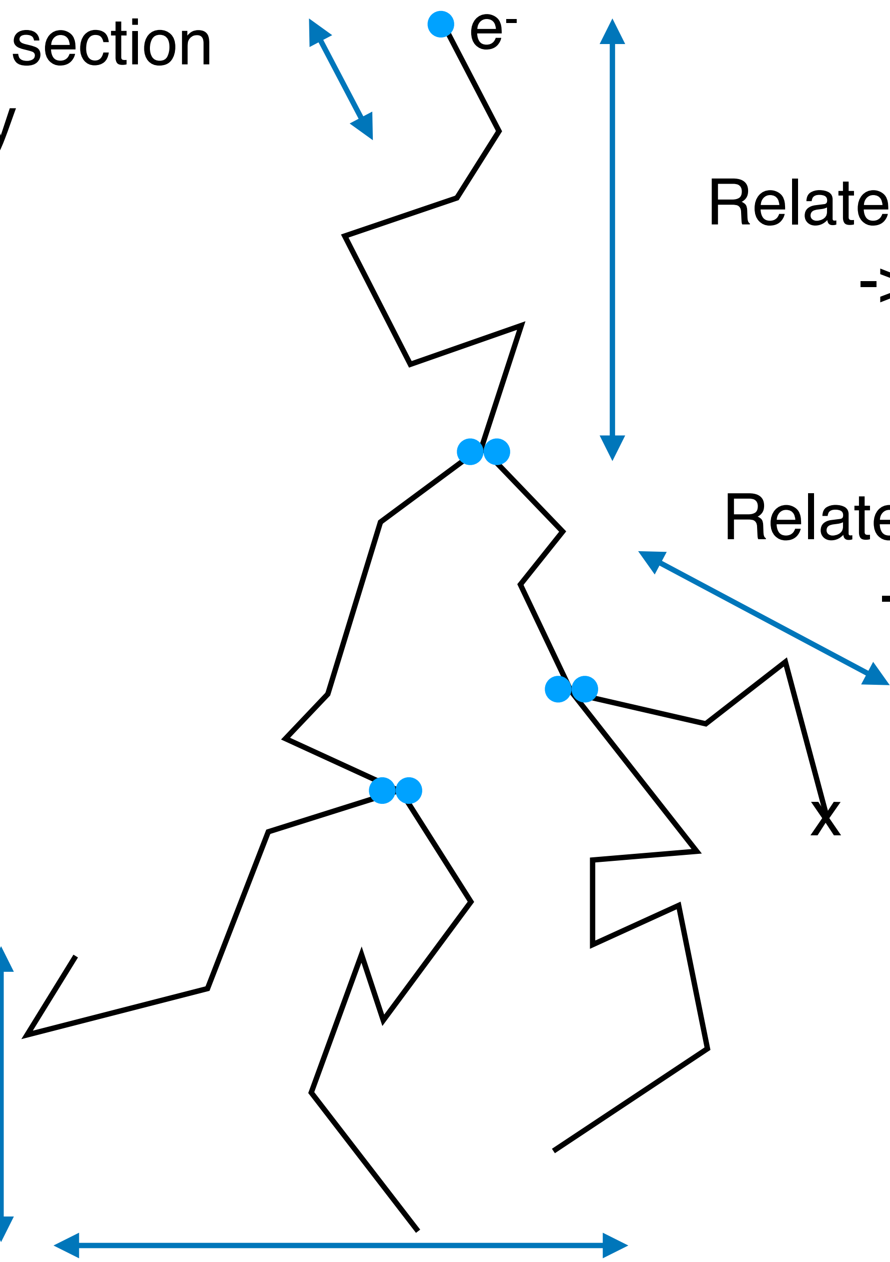
Attack different kind of problems where transient and dynamical interplay is strongly present

Given the right tools, this approach can be powerful and fast

Transport coefficients

Related to total cross section
-> Drift velocity

Related to ionisation cross section
-> Townsend coefficient



E
All coefficients depend
on the electric field

Related to attachment cross section
-> Attachment coefficient

Related to longitudinal diffusion

Related to transversal diffusion

Transport coefficients

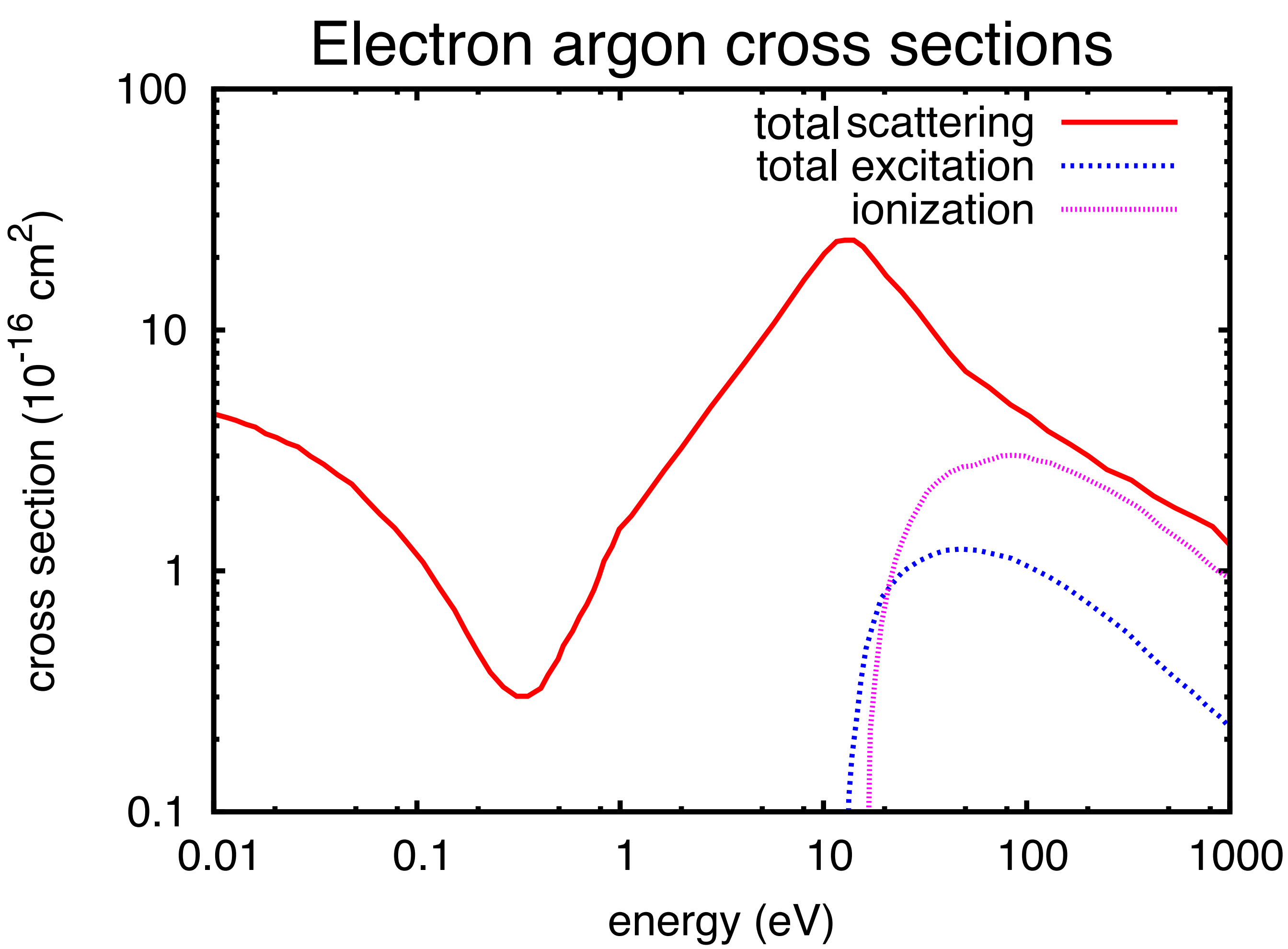
Townsend coefficient **α**:
number of ionised electrons by each drifting electron per unit drift path

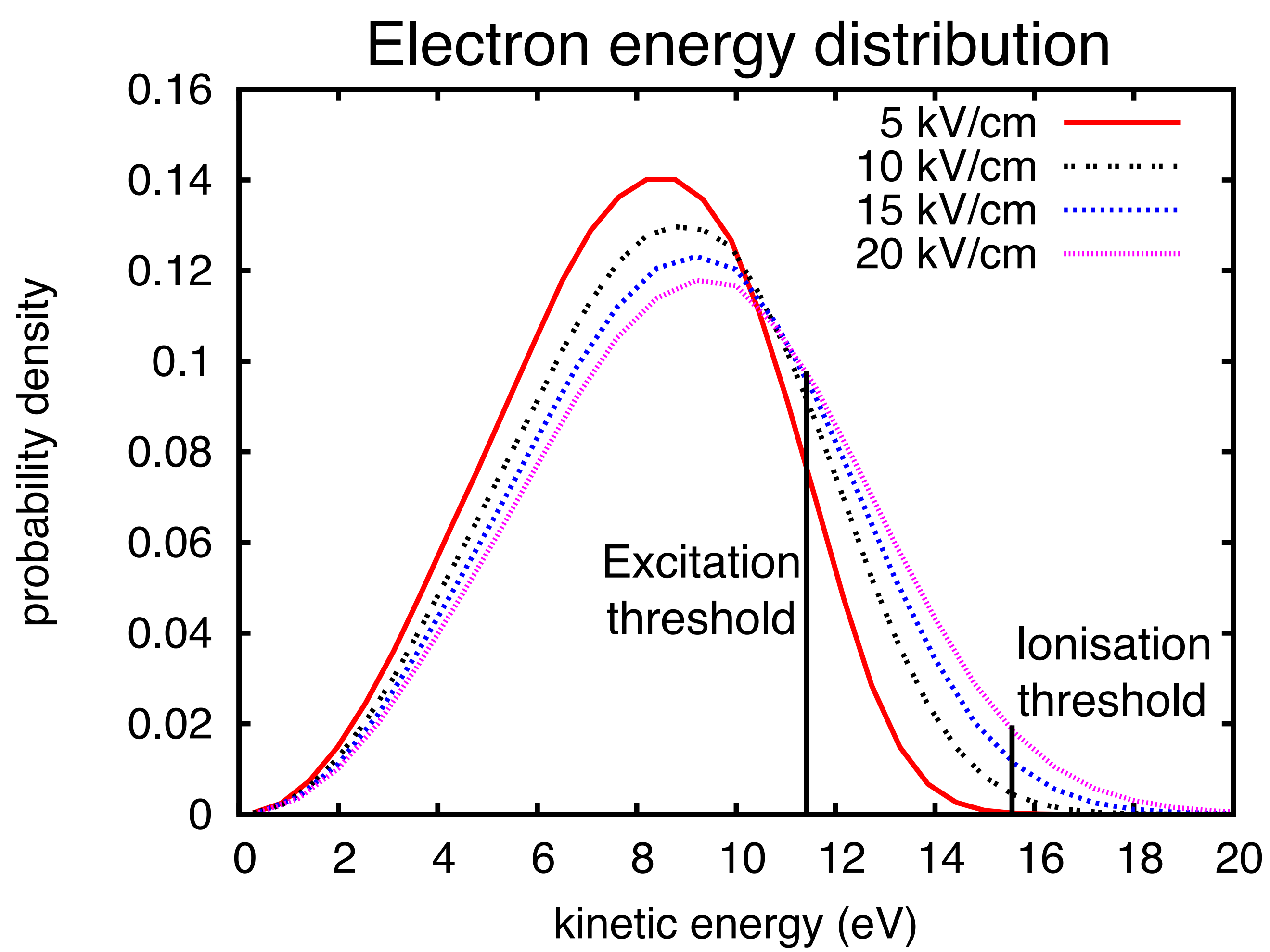
Attachment coefficient **η**:
number of electron captured by electronegative molecules per unit path

Drift velocity **W**

Transversal $\mathbf{D_T}$ and longitudinal $\mathbf{D_L}$ diffusion coefficients with respect to the electric field

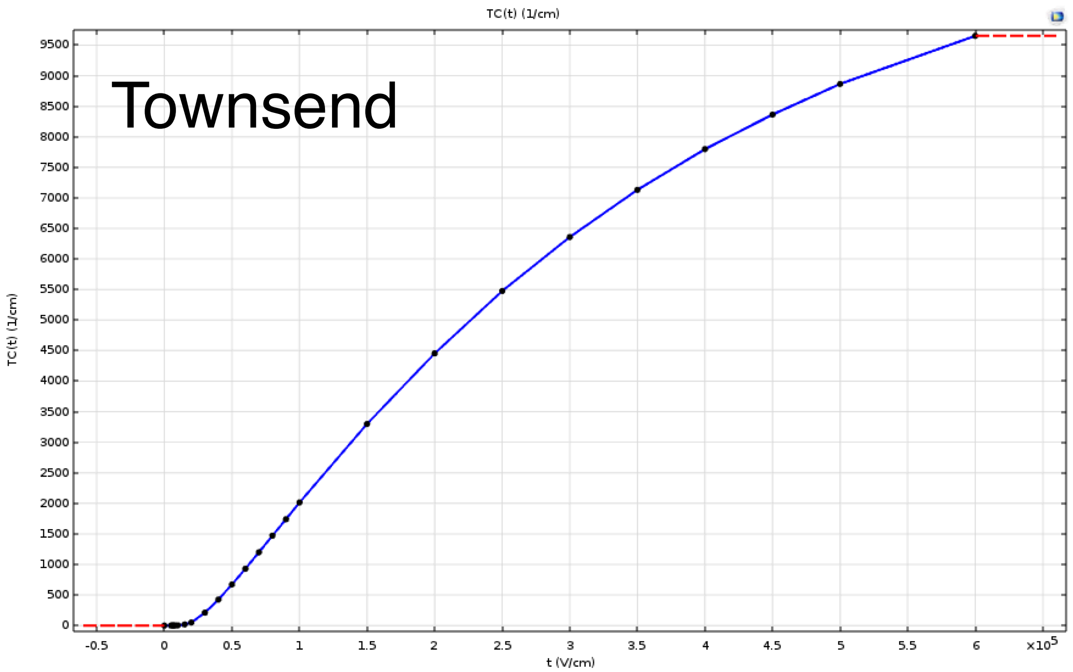
Transport coefficients

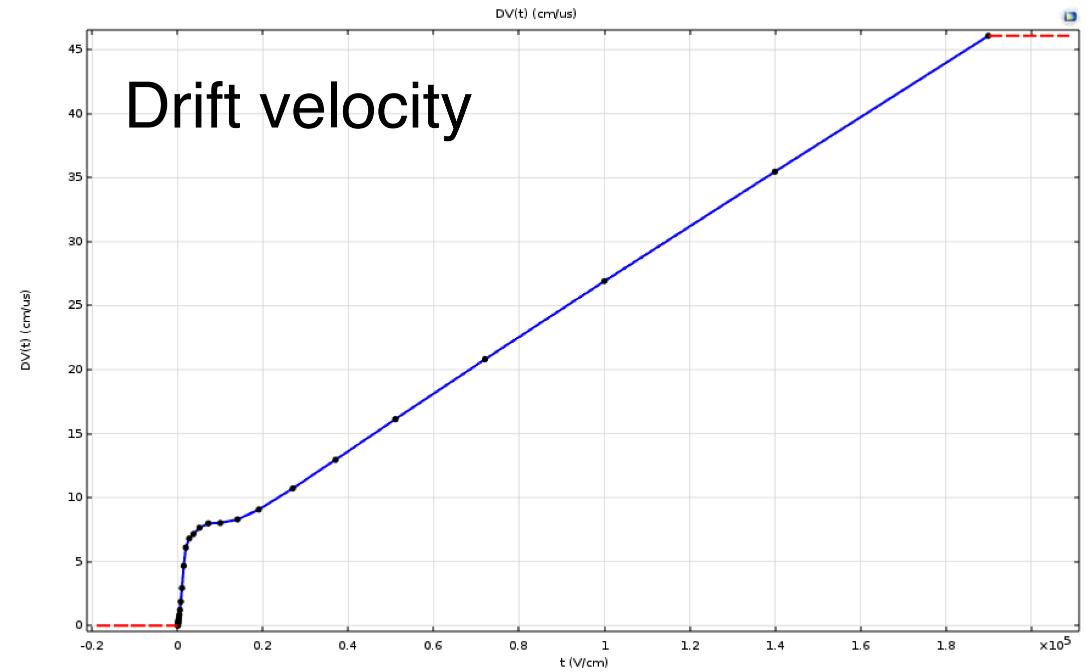


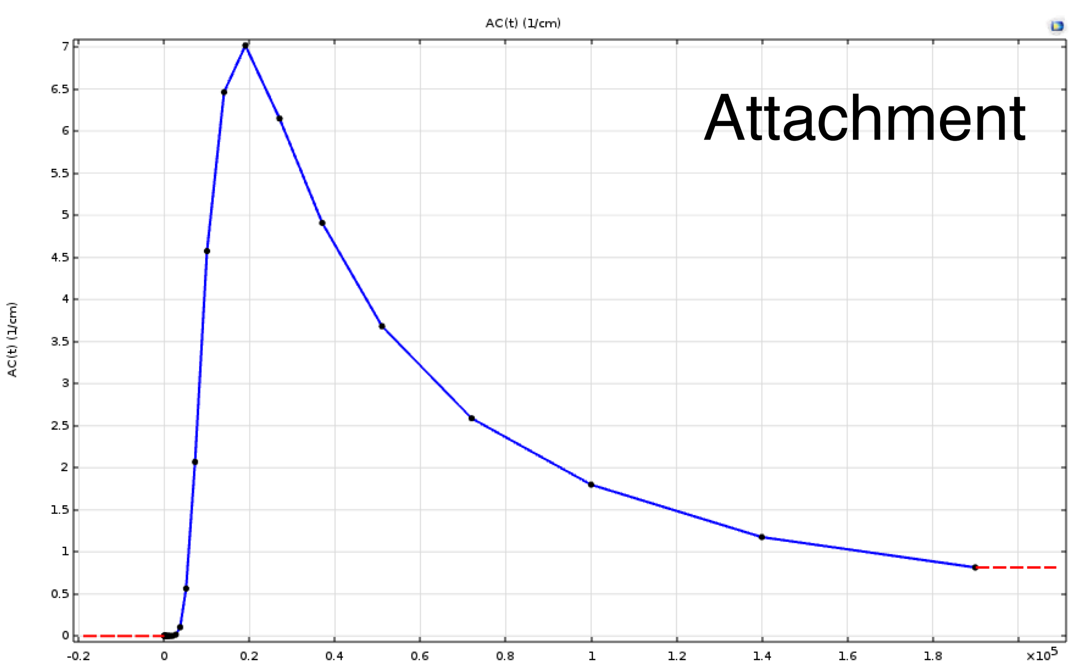


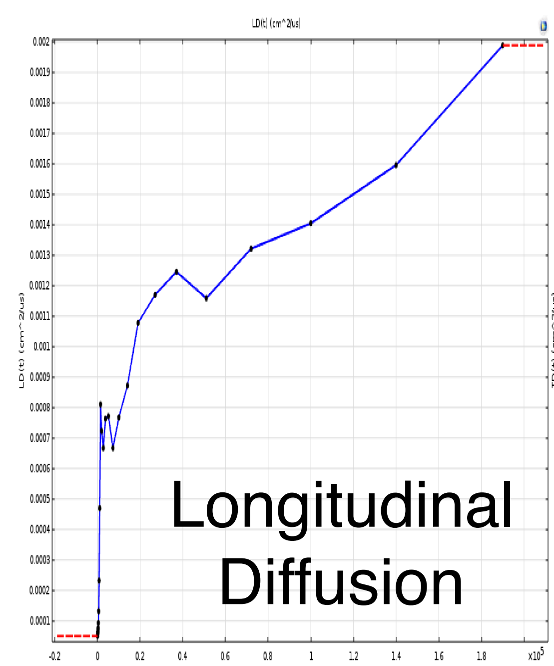
Transport coefficients

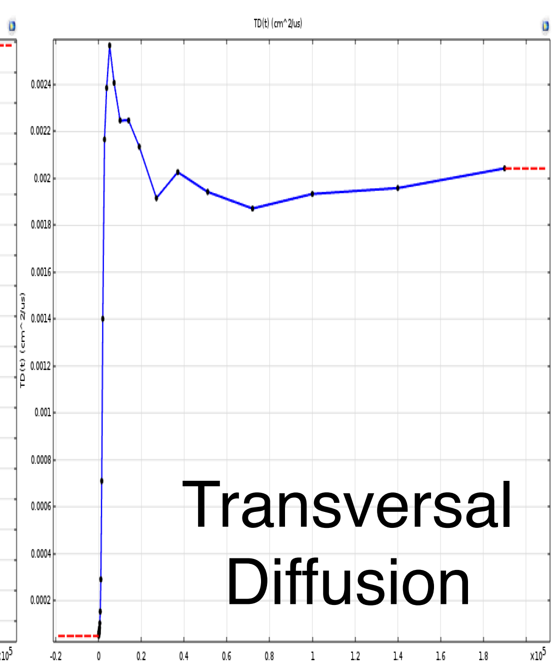
Ar/CO2 70/30 Illustrative purpose only











Electrostatic

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \epsilon \vec{\nabla} V = -\rho$$

V is the unknown

For what concerns the boundary conditions, the most important:

$$V = \phi(\vec{x}, t)$$

The model

$$\vec{\nabla} \cdot \epsilon \vec{\nabla} V = -q_e(\rho_i - \rho_n - \rho_e)$$

$$\frac{\partial \rho_e}{\partial t} = \alpha |\vec{W}_e| \rho_e - \eta |\vec{W}_e| \rho_e - K \rho_i \rho_e - \vec{\nabla} \cdot (\vec{W}_e \rho_e - D_e \vec{\nabla} \rho_e)$$

$$\frac{\partial \rho_i}{\partial t} = \alpha |\vec{W}_e| \rho_e - K \rho_i \rho_e - \vec{\nabla} \cdot (\vec{W}_i \rho_i - D_i \vec{\nabla} \rho_i)$$

$$\frac{\partial \rho_n}{\partial t} = \eta |\vec{W}_e| \rho_e - \vec{\nabla} \cdot (\vec{W}_n \rho_n - D_n \vec{\nabla} \rho_n)$$

The model

If not interested in negative ions

$$\vec{\nabla} \cdot \epsilon \vec{\nabla} V = -q_e(\rho_i - \rho_n - \rho_e)$$

$$\frac{\partial \rho_e}{\partial t} = \alpha |\vec{W}_e| \rho_e - \eta |\vec{W}_e| \rho_e - K \rho_i \rho_e - \vec{\nabla} \cdot (\vec{W}_e \rho_e - D_e \vec{\nabla} \rho_e)$$

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$$\frac{\partial \rho_n}{\partial t} = \eta |\vec{W}_e| \rho_e - \vec{\nabla} \cdot (\vec{W}_n \rho_n - D_n \vec{\nabla} \rho_n)$$

The model

Neglect ion diffusion

$$\vec{\nabla} \cdot \epsilon \vec{\nabla} V = -q_e(\rho_i - \rho_n - \rho_e)$$

$$\frac{\partial \rho_e}{\partial t} = \alpha |\vec{W}_e| \rho_e - \eta |\vec{W}_e| \rho_e - K \rho_i \rho_e - \vec{\nabla} \cdot (\vec{W}_e \rho_e - D_e \vec{\nabla} \rho_e)$$

$$\frac{\partial \rho_i}{\partial t} = \alpha |\vec{W}_e| \rho_e - K \rho_i \rho_e - \vec{\nabla} \cdot (\vec{W}_i \rho_i - D_i \vec{\nabla} \rho_i)$$

$$\frac{\partial \rho_n}{\partial t} = \eta |\vec{W}_e| \rho_e - \vec{\nabla} \cdot (\vec{W}_n \rho_n - D_n \vec{\nabla} \rho_n)$$

The model

Neglect recombination

$$\vec{\nabla} \cdot \epsilon \vec{\nabla} V = -q_e(\rho_i - \cancel{\rho_n} - \rho_e)$$

$$\frac{\partial \rho_e}{\partial t} = \alpha |\vec{W}_e| \rho_e - \eta |\vec{W}_e| \rho_e - K \cancel{\rho_i} \rho_e - \vec{\nabla} \cdot (\vec{W}_e \rho_e - D_e \vec{\nabla} \rho_e)$$

$$\frac{\partial \rho_i}{\partial t} = \alpha |\vec{W}_e| \rho_e - K \cancel{\rho_i} \rho_e - \vec{\nabla} \cdot (\vec{W}_i \rho_i - \cancel{D_i \vec{\nabla}} \rho_i)$$

$$\frac{\partial \rho_n}{\partial t} = \eta |\vec{W}_e| \rho_e - \vec{\nabla} \cdot (\vec{W}_n \rho_n - D_n \vec{\nabla} \rho_n)$$

Note Absence of gas photo-ionisation

How to solve it?

COMSOL:

Finite Element Analysis software able to find an approximate solution for a coupled system of PDEs on an (almost) arbitrary (3D, 2D, and 1D) mesh

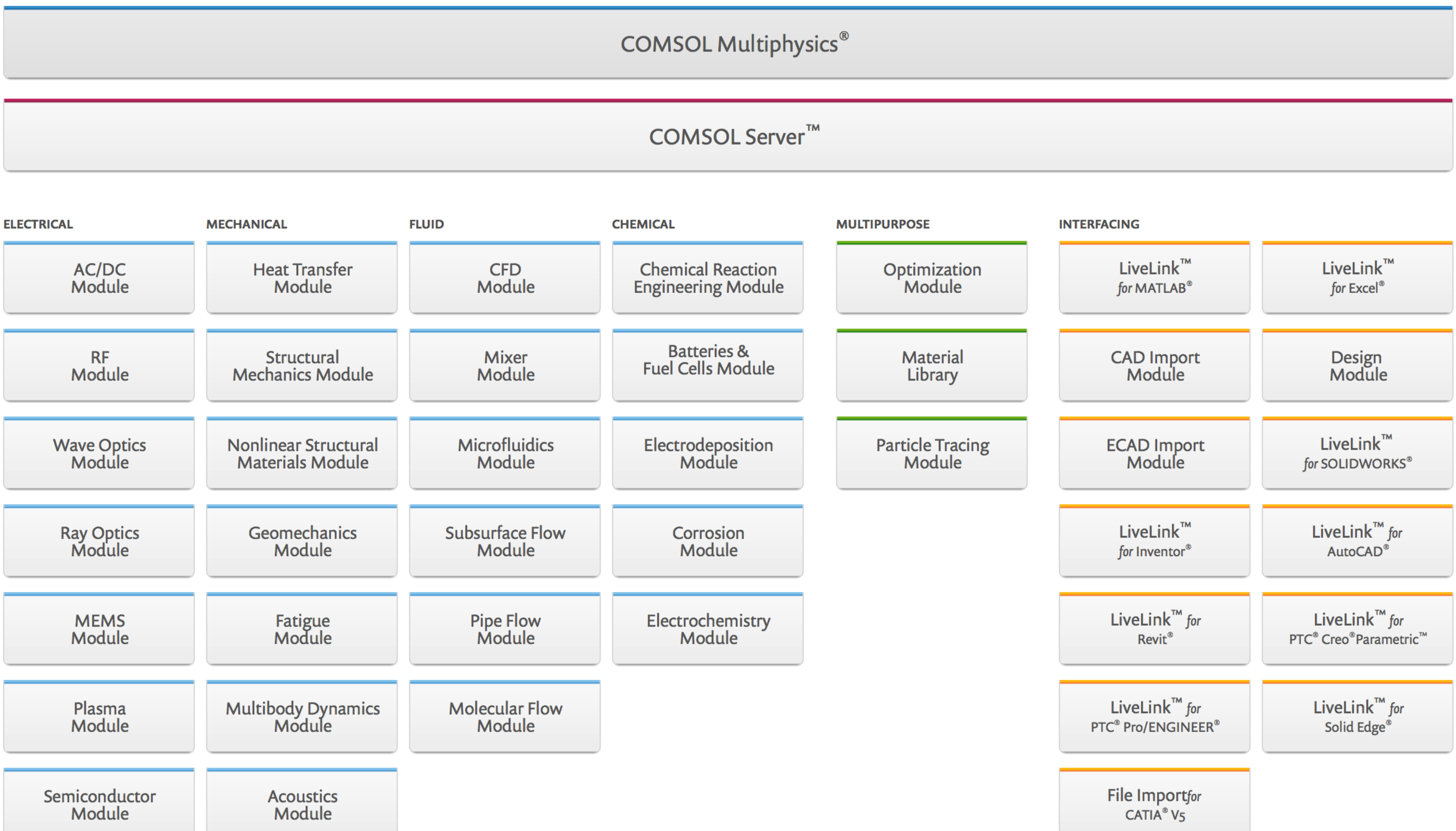
Coefficients are arbitrary

convection
source

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f$$

mass
damping
diffusion
conservative convection
flux source
absorption

Multiphysics



But there's no gaseous detector model…

Its strength

Link different physics problems (via the proper choice of the coefficients)

Example:
Compute the thermal deformation of a resistor (Joule effect, heat transfer, mechanical deformation, …)

Perfect to compute electric fields that depends on charges that move depending on the electric field

Example 1
Amplification

Avalanche

Exact solution in an infinite volume with uniform and constant electric field

$$\frac{\partial \rho_e}{\partial t} = D_e \frac{\partial^2 \rho_e}{\partial z^2} - v_e \frac{\partial \rho_e}{\partial z} + \alpha v_e \rho_e$$

$$\rho_e(z,t) = N_0 \frac{e^{\alpha v_e t} e^{-(z - v_e t)^2/(4 D_e t)}}{\sqrt{4\pi D_e t}} \text{ for } t > 0$$

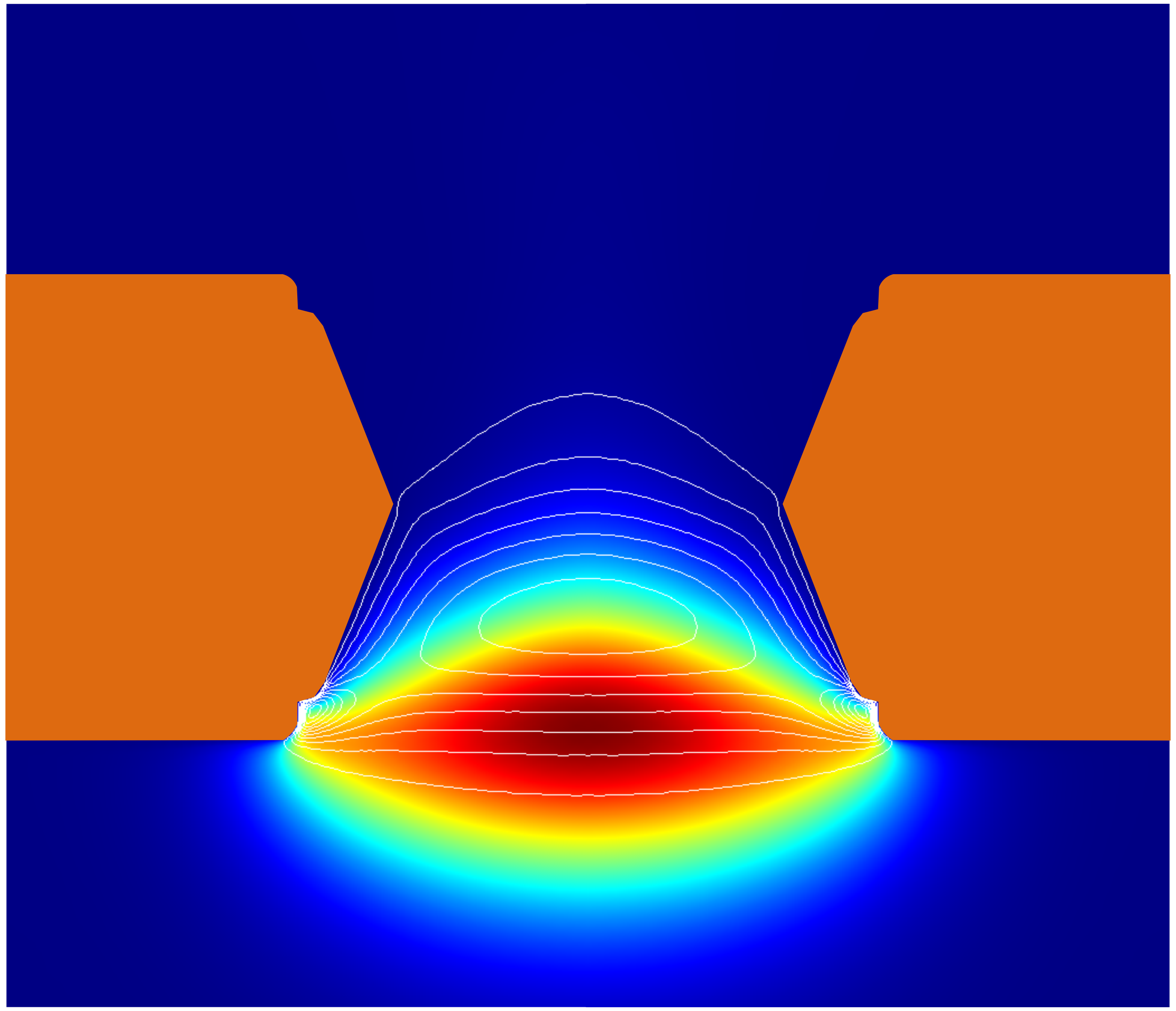
$$\frac{\partial \rho_i}{\partial t} = -v_i \frac{\partial \rho_i}{\partial z} + \alpha v_e \rho_e$$

$$\rho_i(z,t) = \int_0^t \alpha v_e N_0 \frac{e^{\alpha v_e s} e^{-(z - v_e s - v_i (t-s))^2/(4 D_e s)}}{\sqrt{4\pi D_e s}} ds \text{ for } t > 0$$

Exponential growing Gaussian
moving at a constant speed
with the standard deviation proportional to sqrt(t)

Avalanche

3D approximated with axis-symmetric 2D often used
3D model possible, but solution may be very time consuming



Example 2
Signal induction

Signal induction

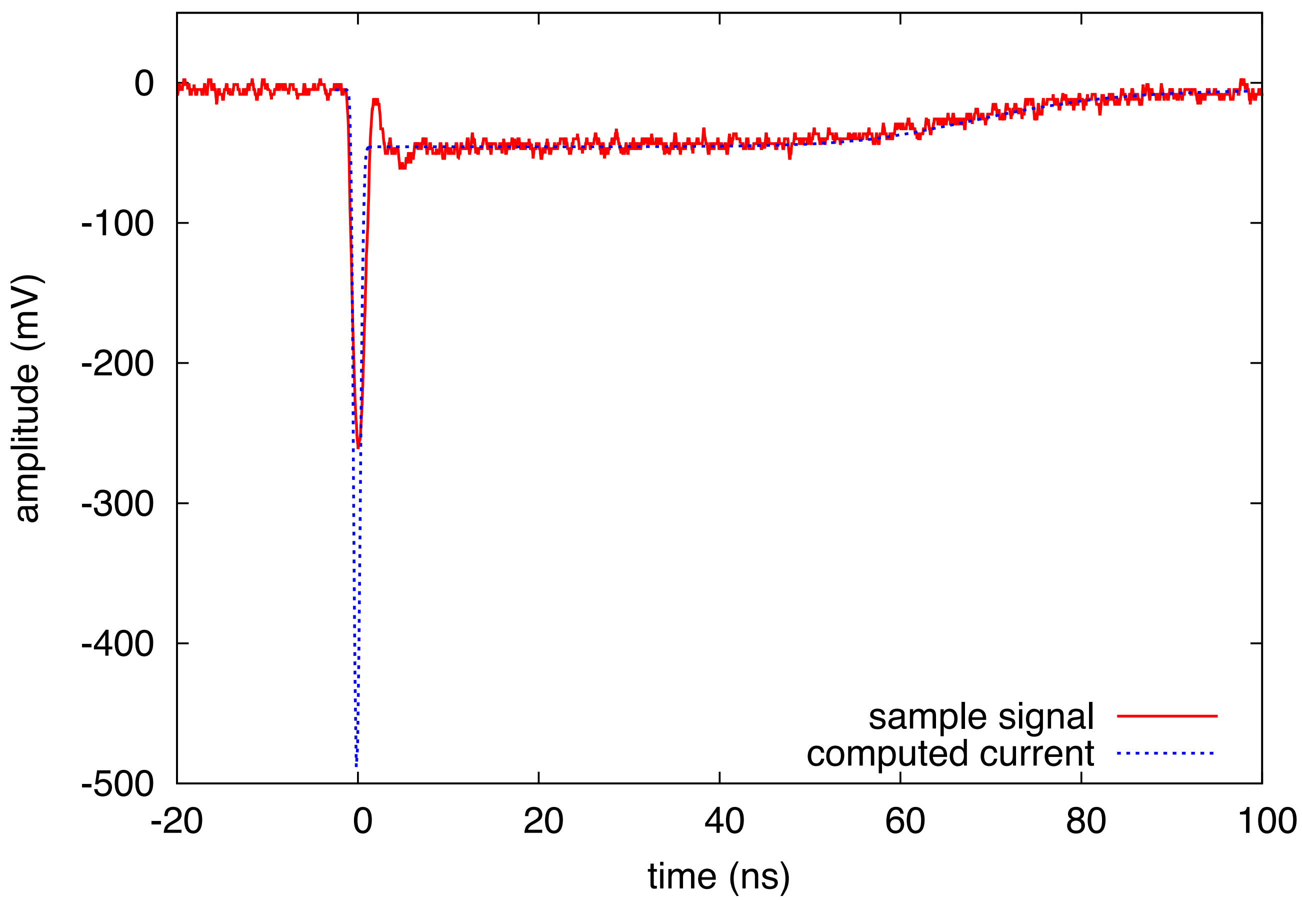
Compute the charge distribution movement
Compute the weighting fields
Compute the induced current

$$I = \sum_i \int \rho_i \vec{v_i} \cdot \vec{W}$$

Add the network component

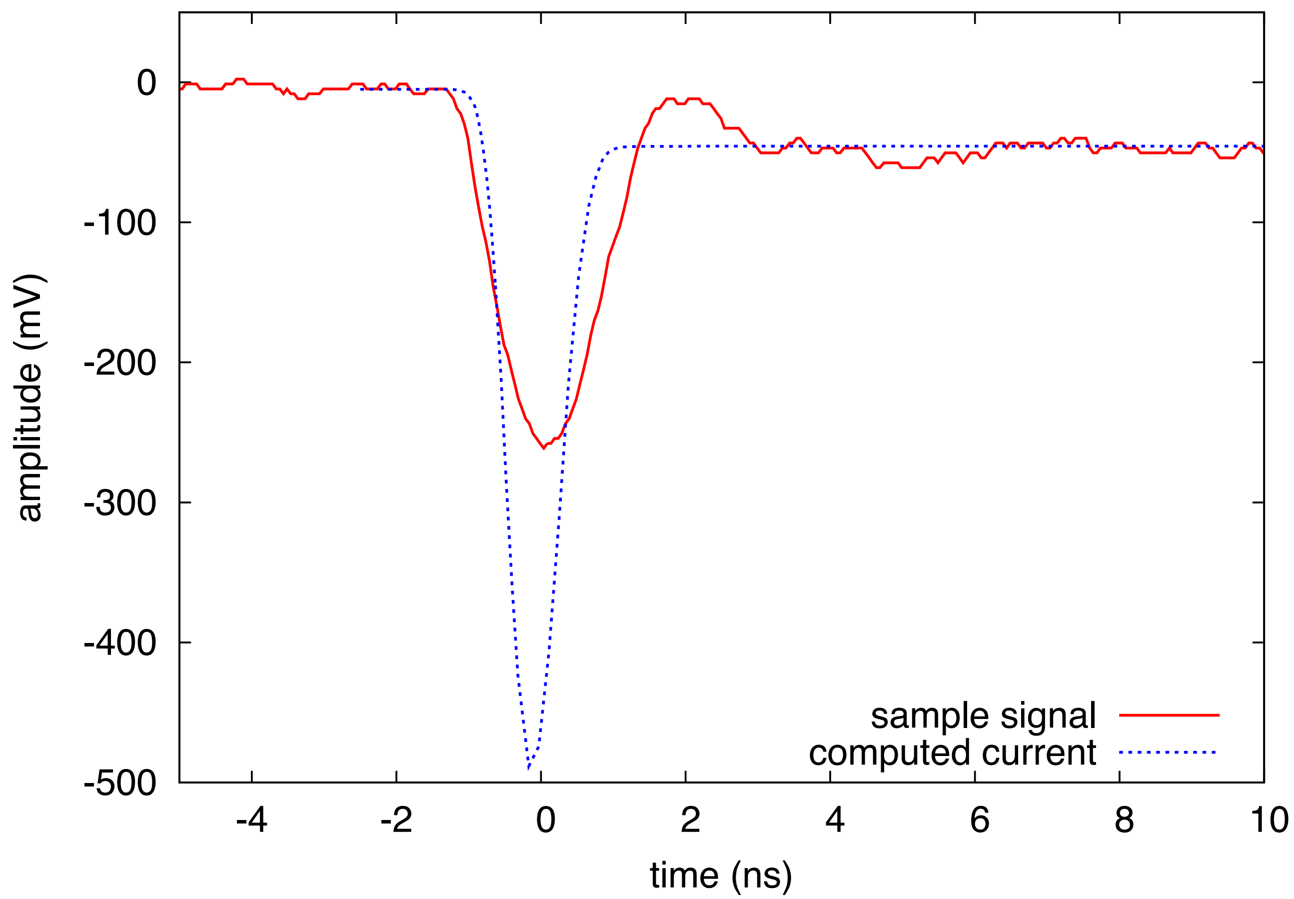
Signal induction

3D computation

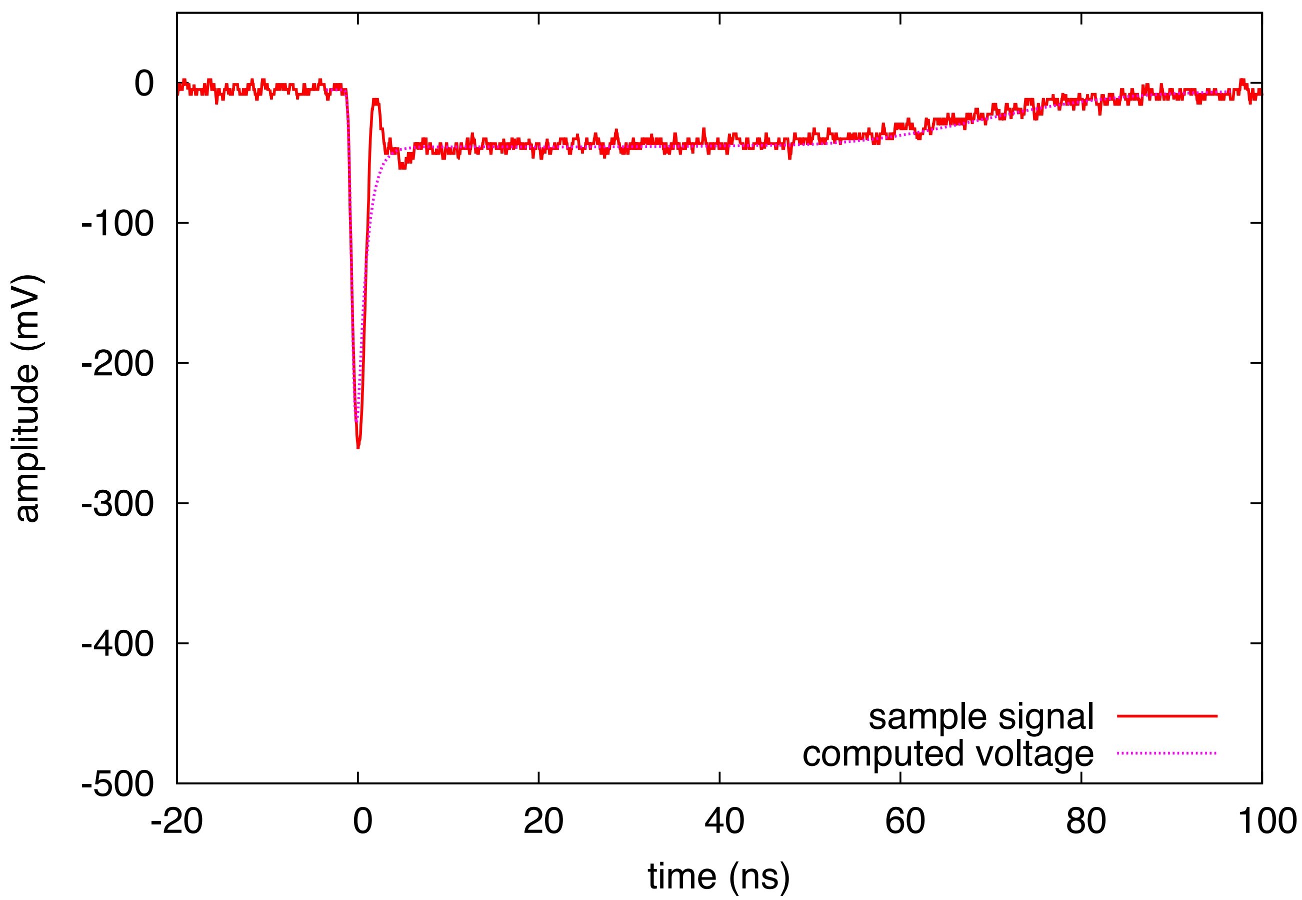


Ion velocity tuned to match the tail

Current over 50 Ohm resistor

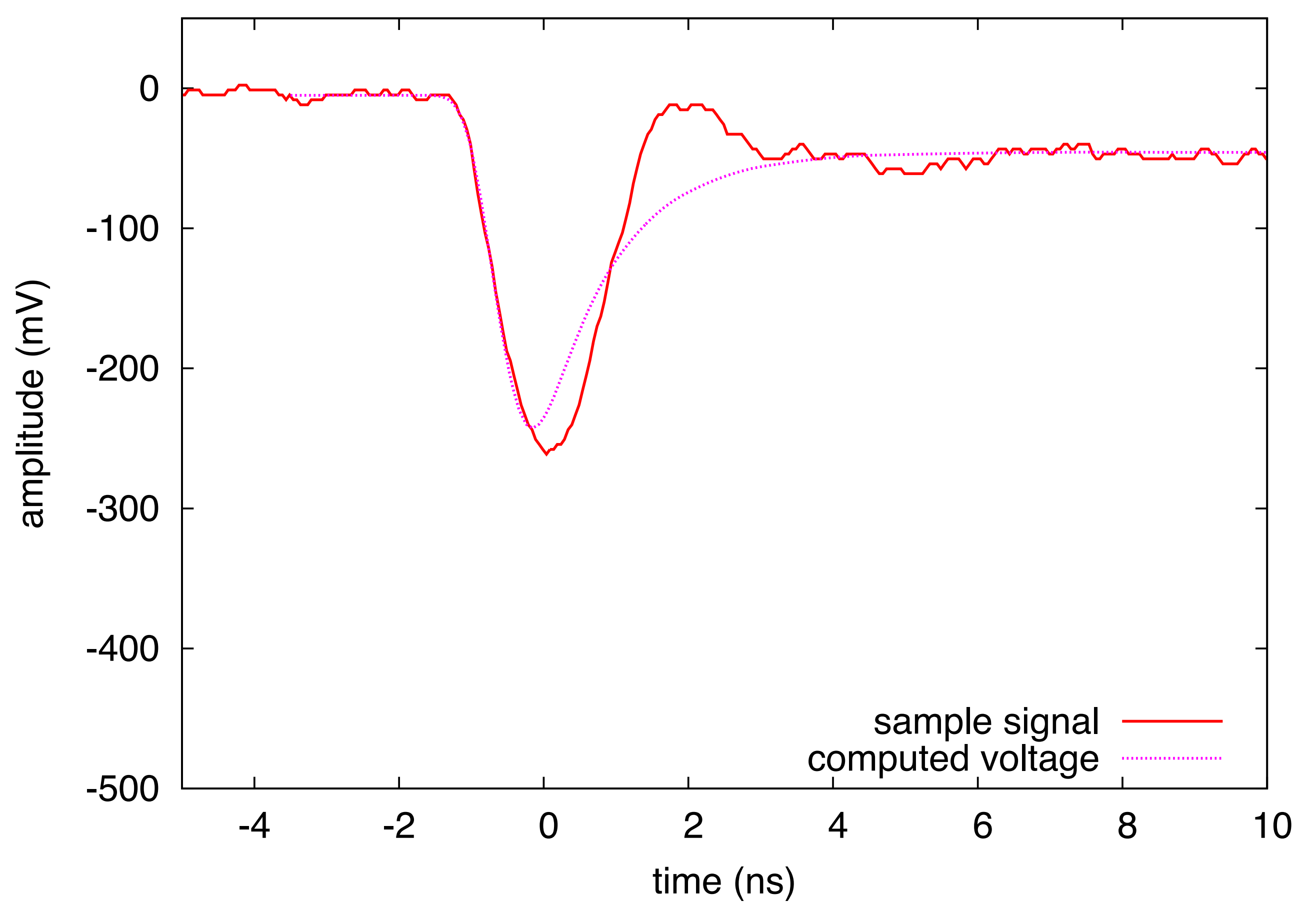


Signal induction

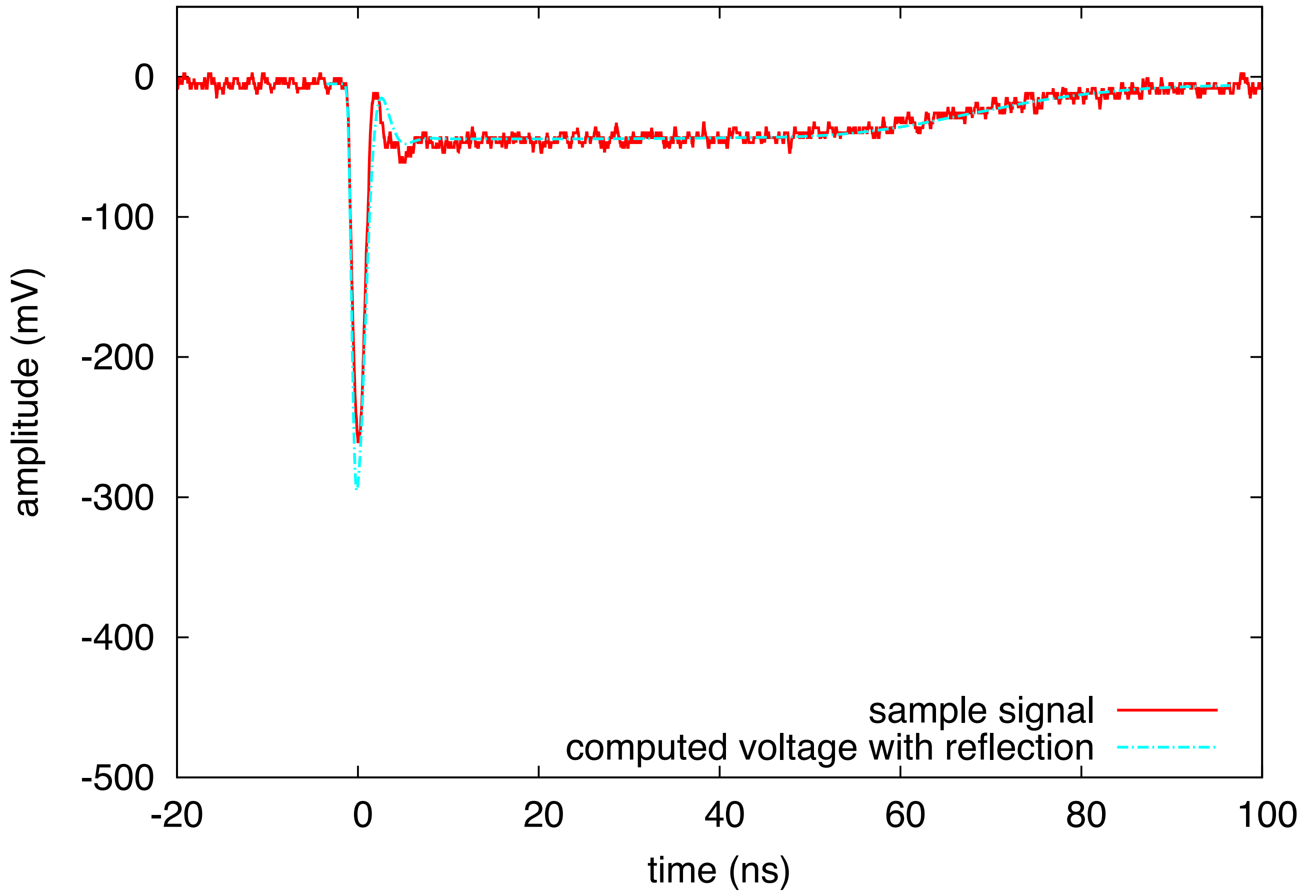


R = 50 Ohm, C = 19 pF

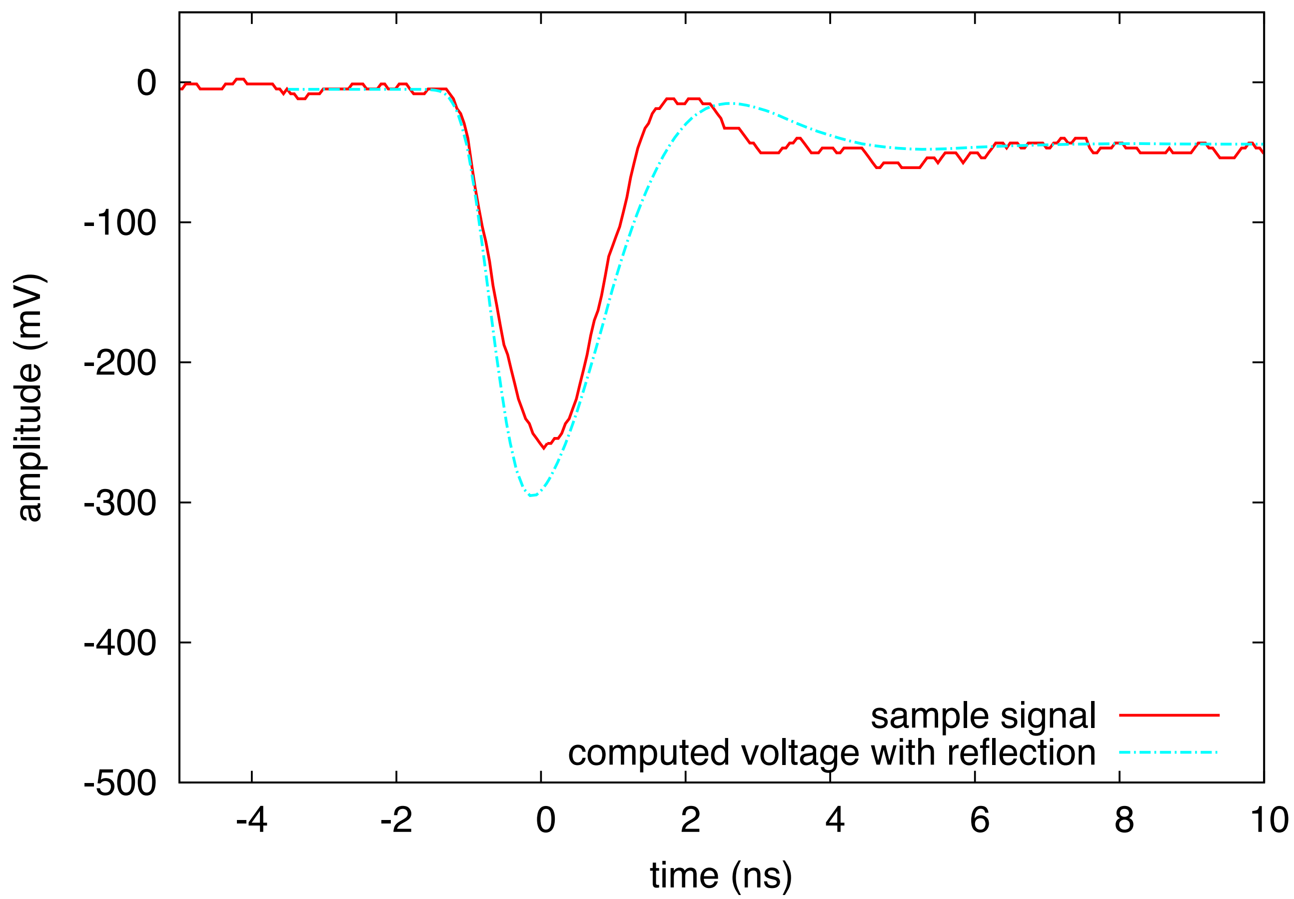
Amplitudes and rise-time match



Signal induction

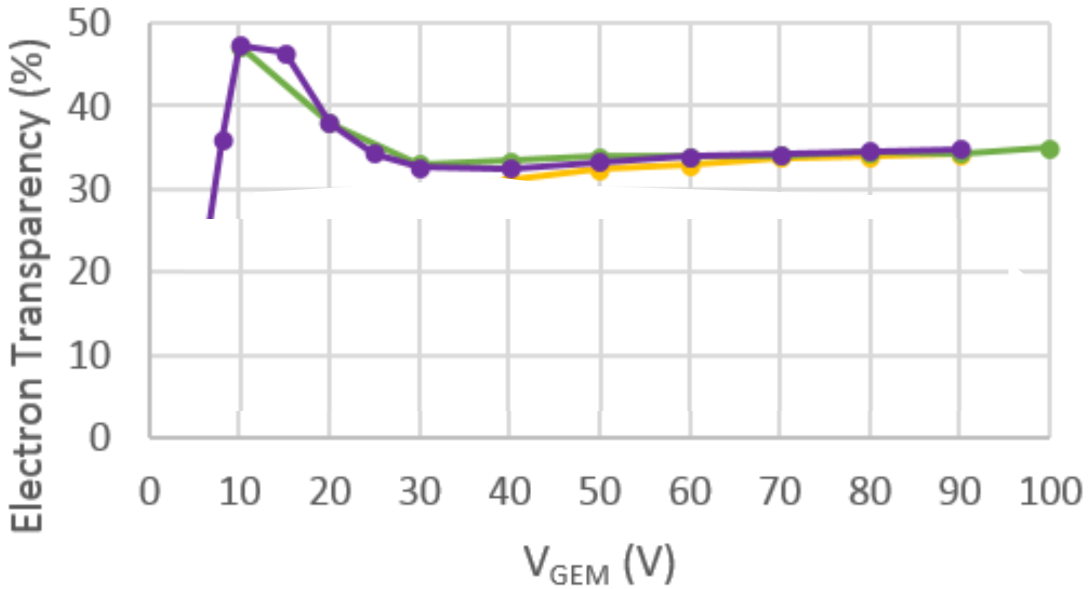


With an ad hoc reflection…

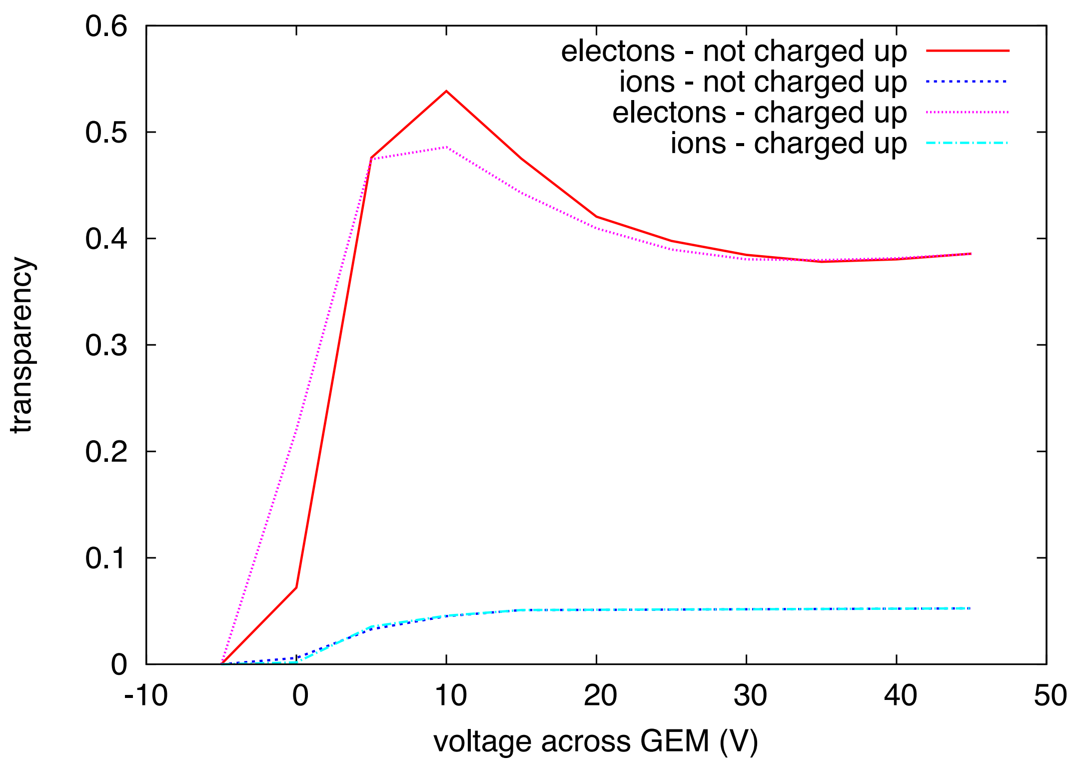


Example 3
Transparency

GEM



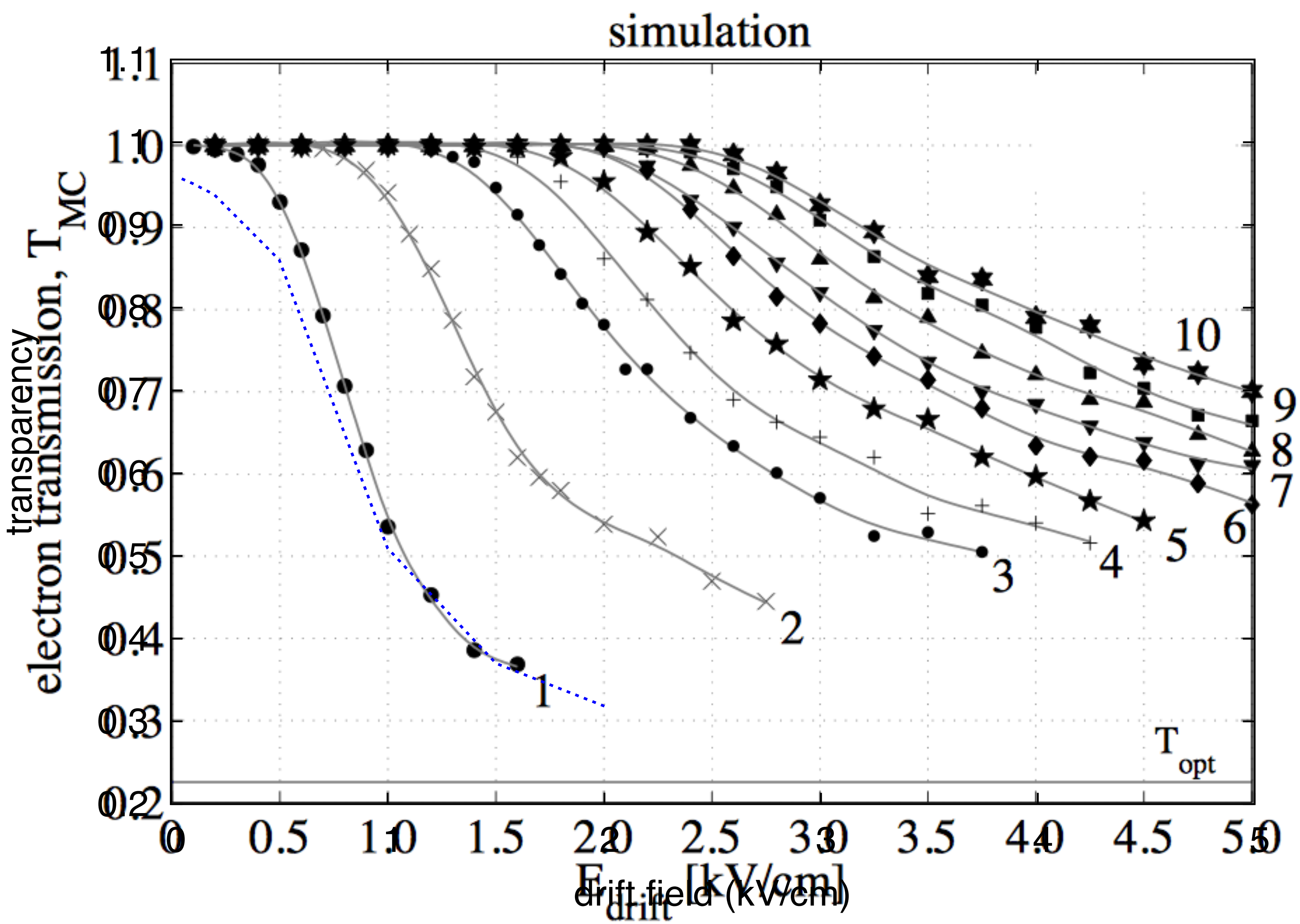
E_{D1} = 50V/cm
E_{D2} = 1kV/cm



E_{D1} = 100V/cm
E_{D2} = 2kV/cm

MM

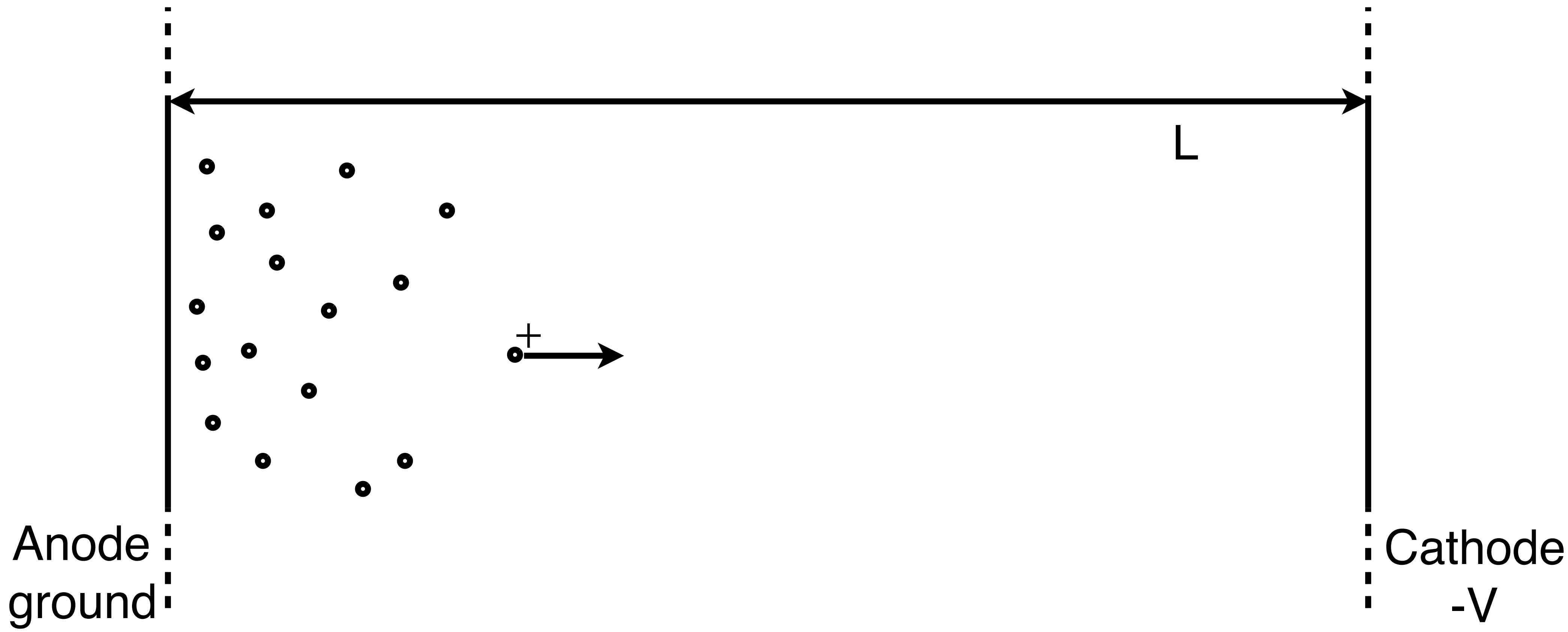
Xe/TMA example



Example 4
Space charge

Now the model becomes really useful:
circular dependence of the charge distributions on
the coefficients that depend on the electric field and
the electric field that depends on the charge distributions

Simplified problem



Infinite parallel plates at distance L with a potential difference of ΔV

At $t = 0$ uniform electric field of $E_0 = \Delta V/L$

Positive ions generated at the anode at a constant and uniform flux R

Ions moving towards the cathode at speed $v = \mu E$

Actual electric field E modified by the charge distribution

Analytical

$$|\vec{v}| = \mu|\vec{E}| = \mu E_z$$

$$R = \rho v_\perp = \rho|\vec{v}|$$

$$\rho/\epsilon = \vec{\nabla}\cdot\vec{E} = \frac{dE_z}{dz}$$

$$R = \epsilon\mu\frac{dE_z}{dz}E_z$$

$$dz = \frac{\epsilon\mu}{R}E_z dE_z$$

$$z = \frac{\epsilon\mu}{R}E_z^2/2 - z_0$$

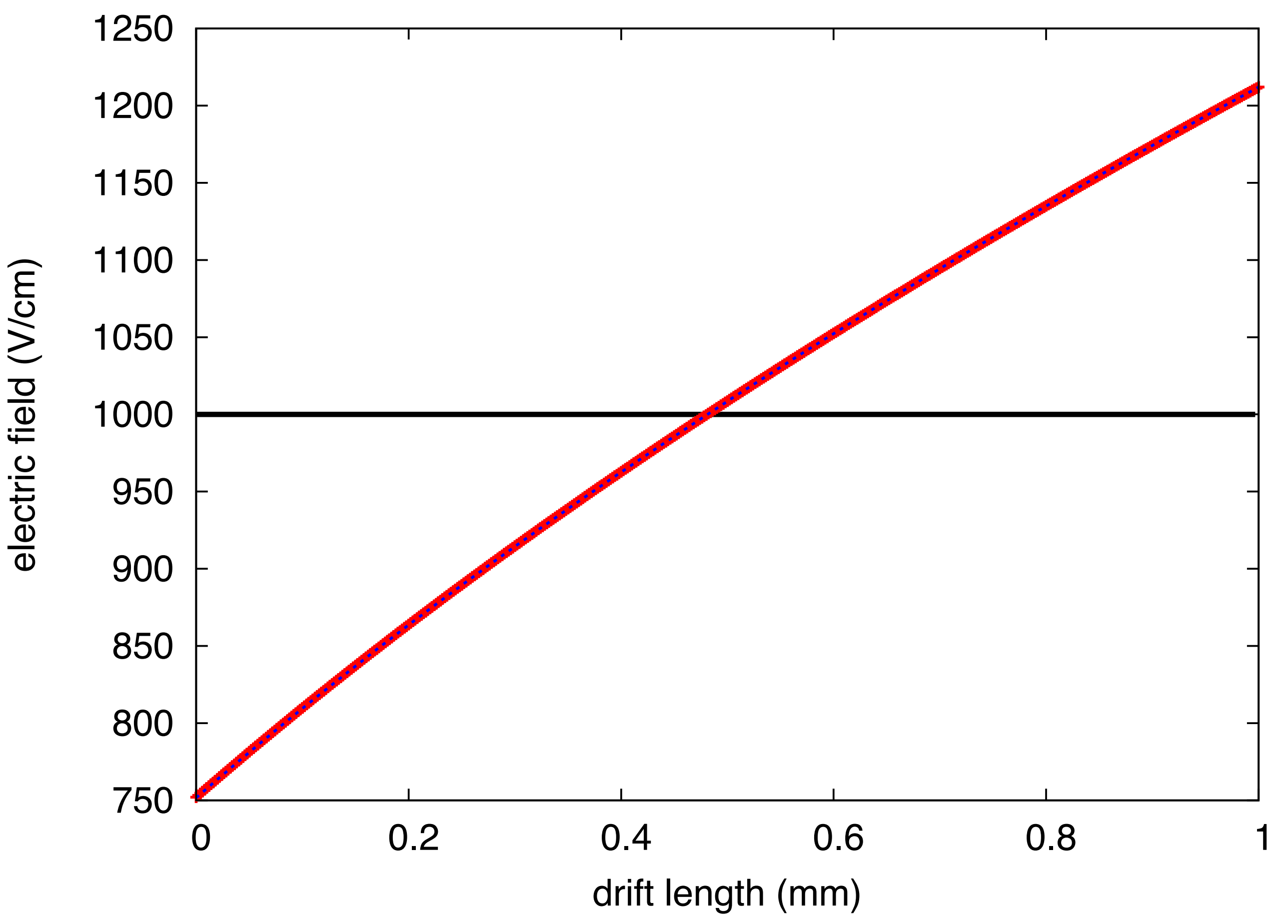
$$E_z = \sqrt{\frac{2R(z+z_0)}{\epsilon\mu}}$$

$$\rho = \epsilon\frac{dE_z}{dz} = \sqrt{\frac{\epsilon R}{2\mu(z+z_0)}}$$

$$\Delta V = \int_0^L E_z dz = \sqrt{\frac{8R}{9\epsilon\mu}}((L+z_0)^{3/2} - z_0^{3/2})$$

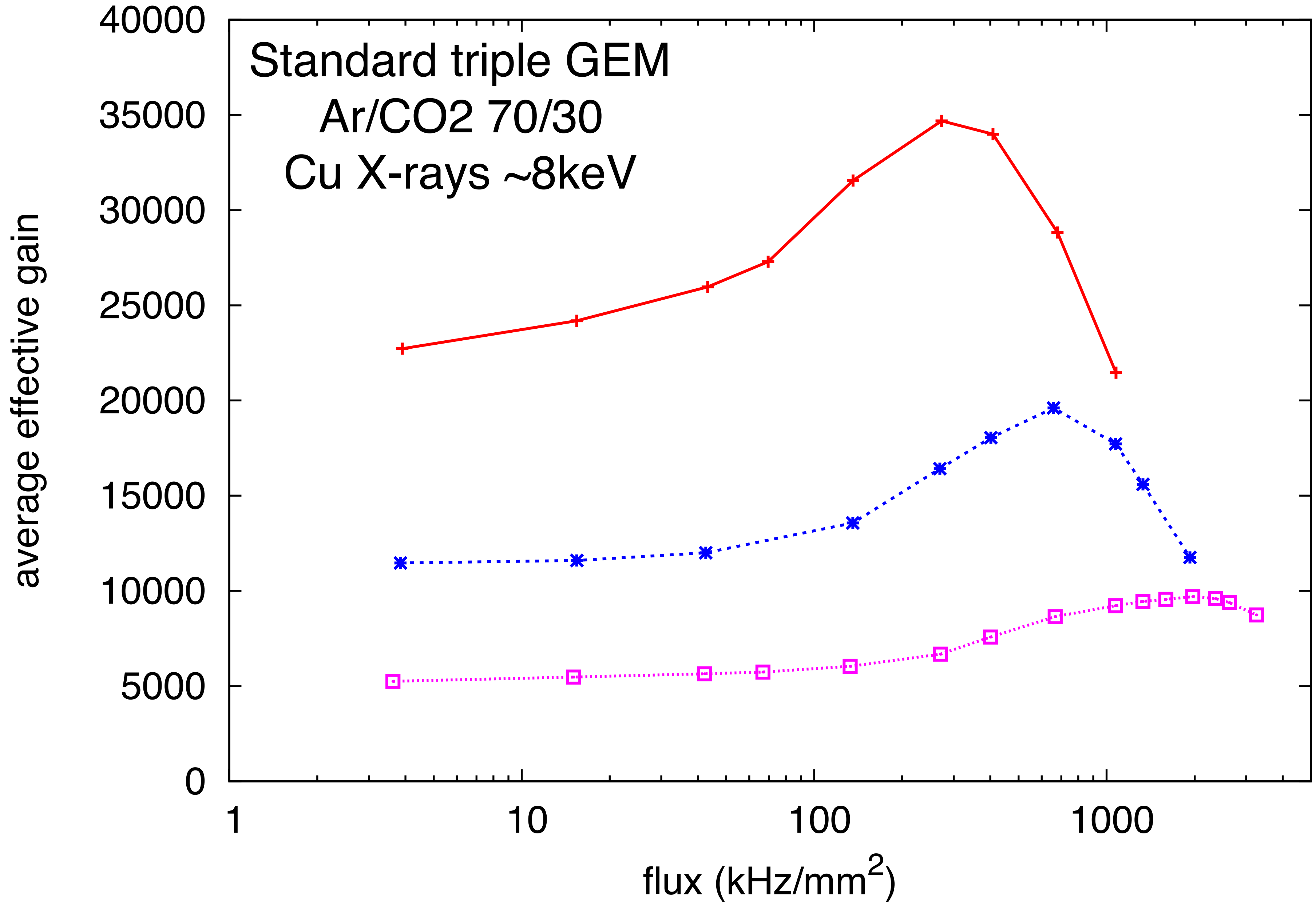
$$R_{max} = \frac{9\epsilon\mu E_0^2}{8L}$$

$$E_{min} = \sqrt{\frac{8RL}{9\epsilon\mu}}$$



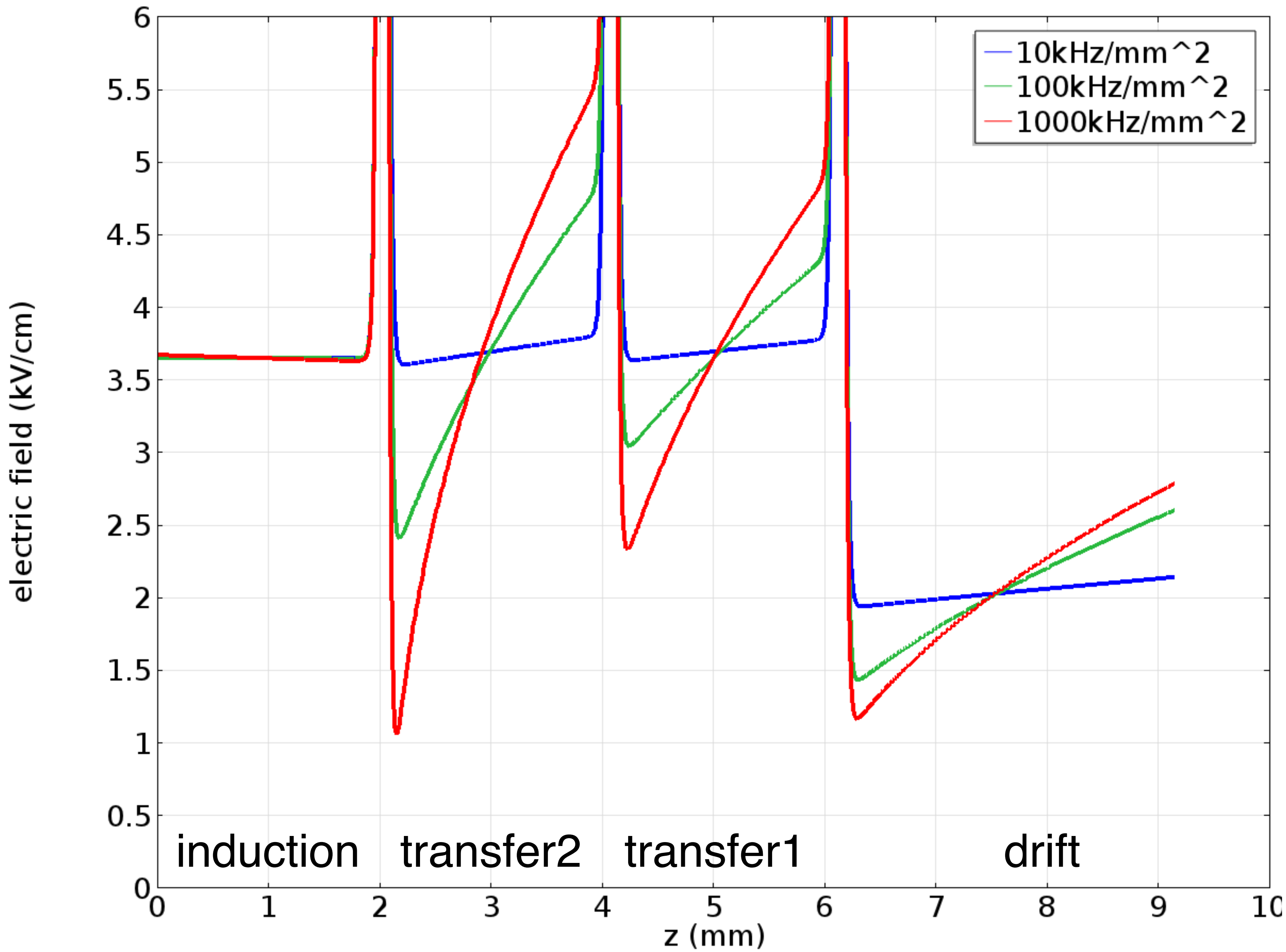
R = 0.16 nA/mm2
μ = 4×10-5 cm2/us/kV
L = 1 mm
E0 = 1 kV/cm

The measurement

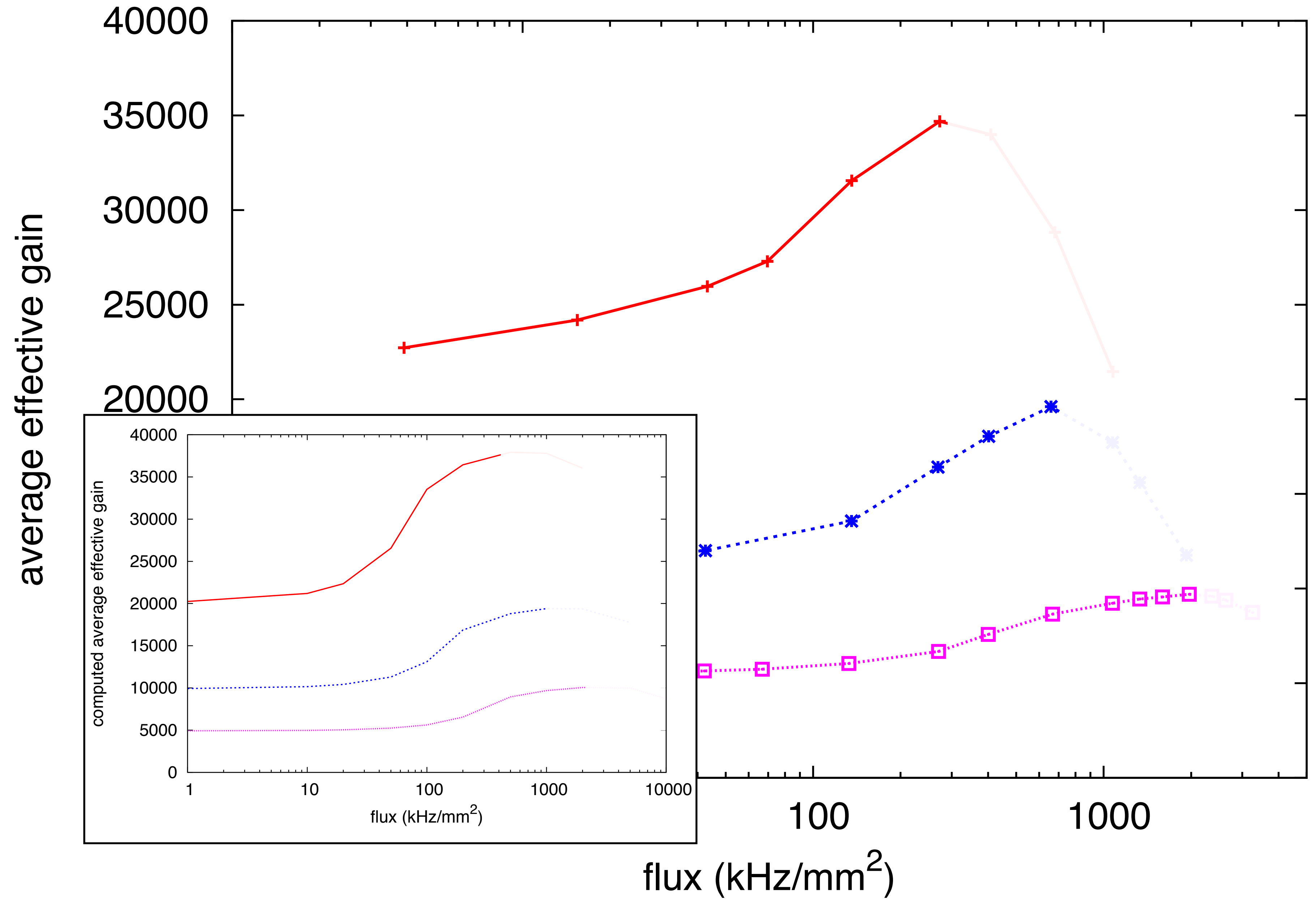


Field modification

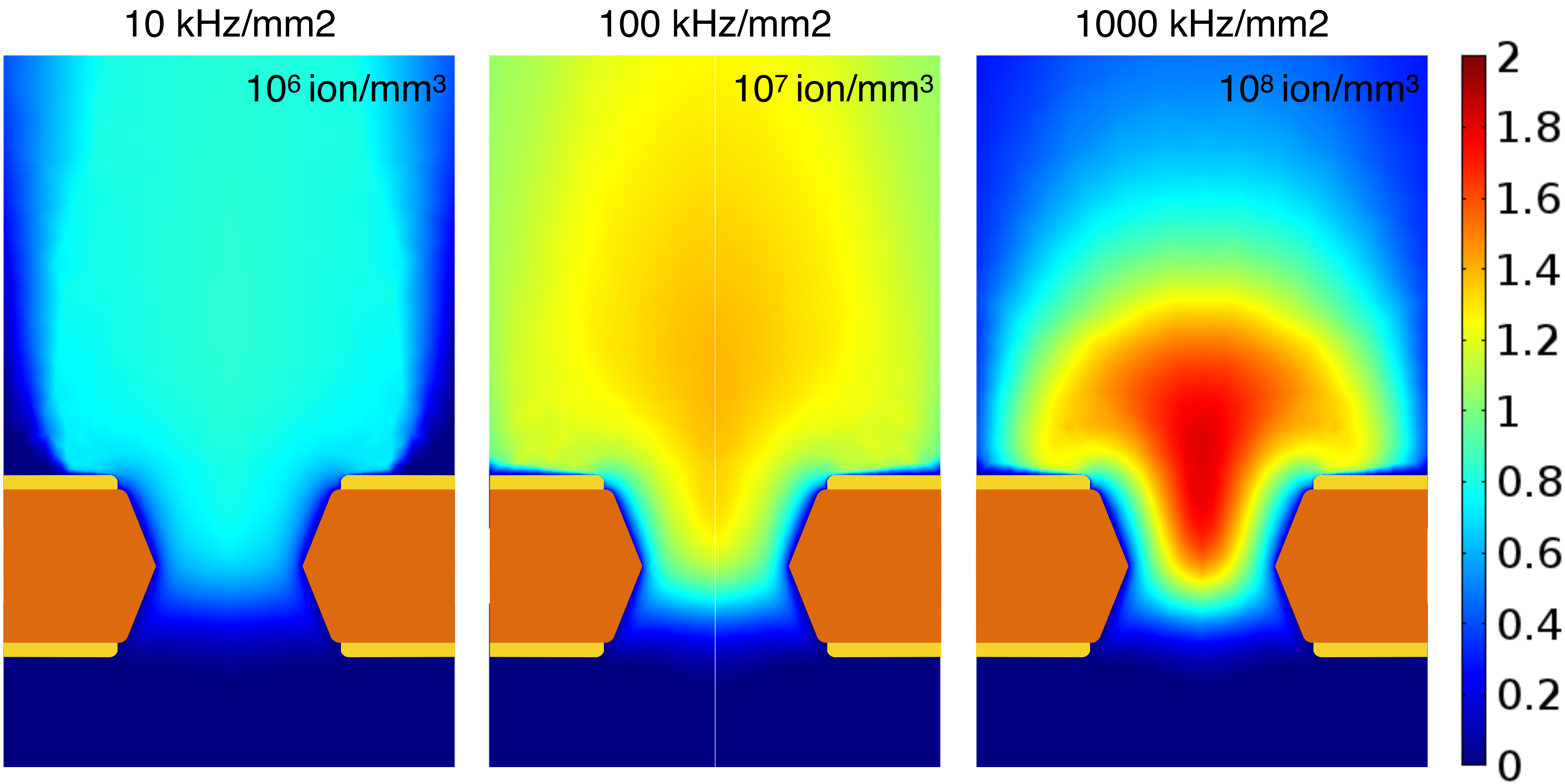
Triple GEM, aligned single holes, axis-symmetric approximation



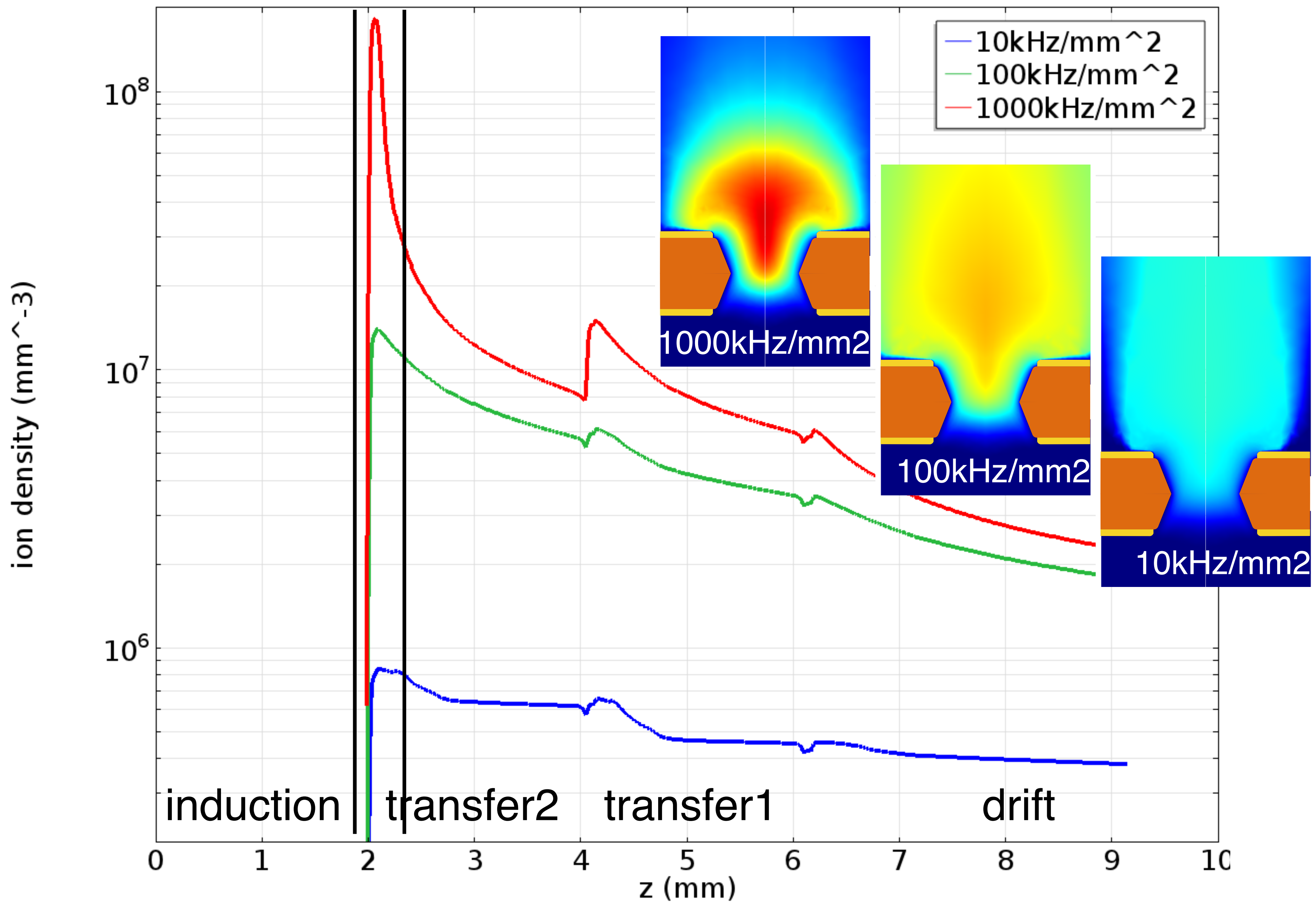
First comparison



In the hole

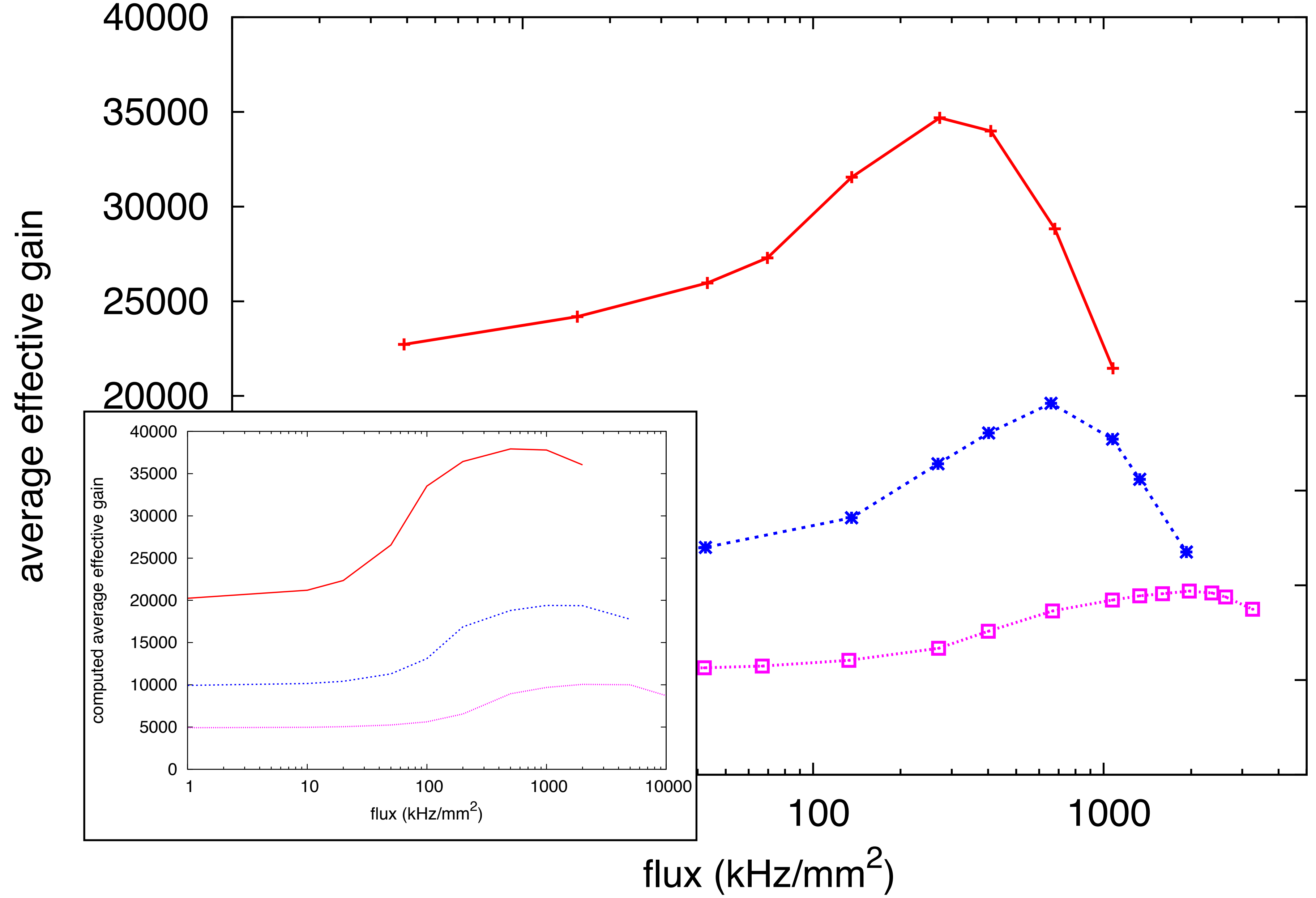

10 kHz/mm2
100 kHz/mm2
1000 kHz/mm2
10⁶ ion/mm³
10⁷ ion/mm³
10⁸ ion/mm³
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

In the hole



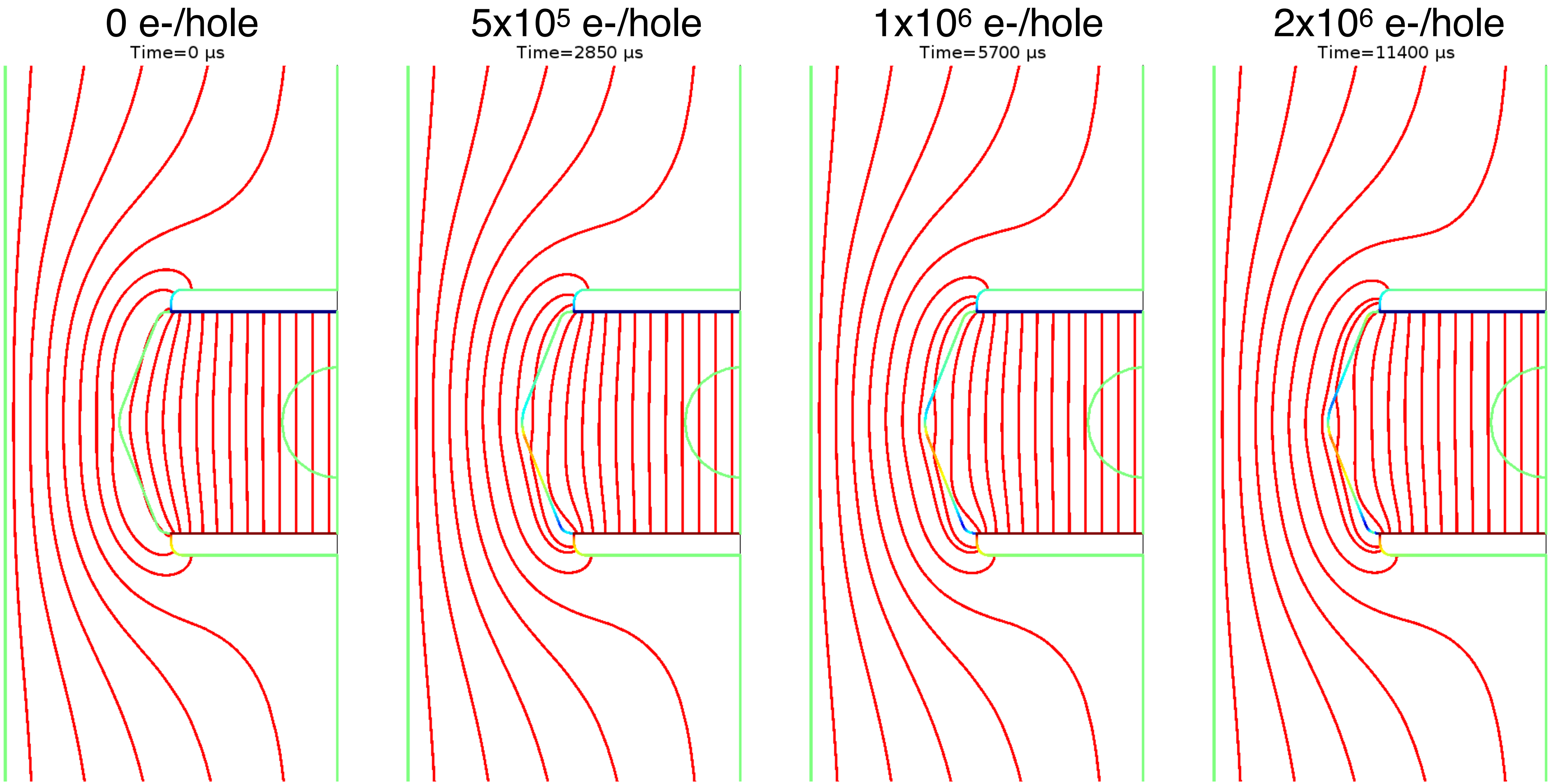
10kHz/mm^2
100kHz/mm^2
1000kHz/mm^2
1000kHz/mm2
100kHz/mm2
10kHz/mm2
ion density (mm^-3)
10^8
10^7
10^6
induction
transfer2
transfer1
drift
0
1
2
3
4
5
6
7
8
9
10
z (mm)

Final comparison



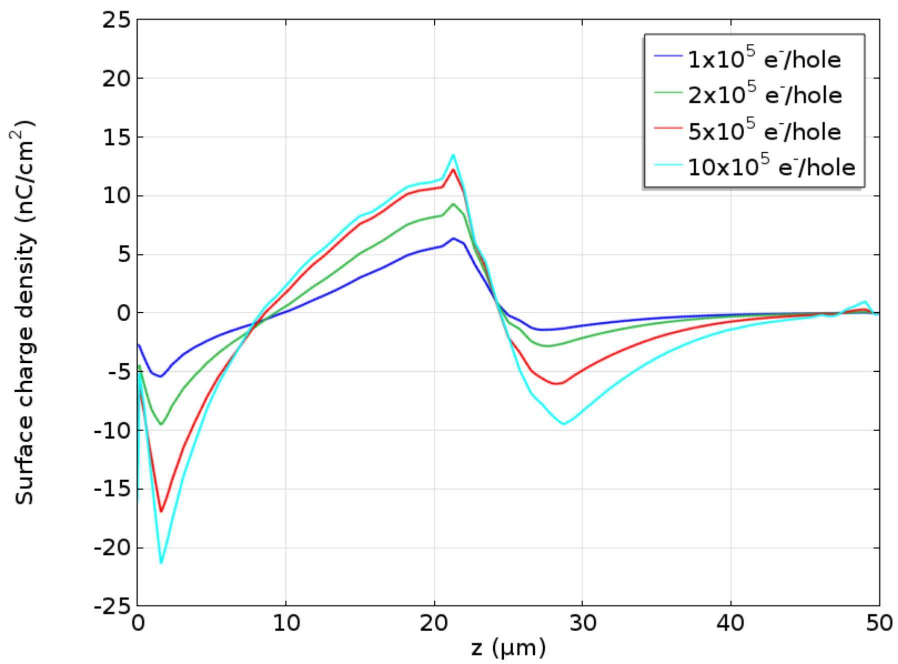
Example 5
Charging up

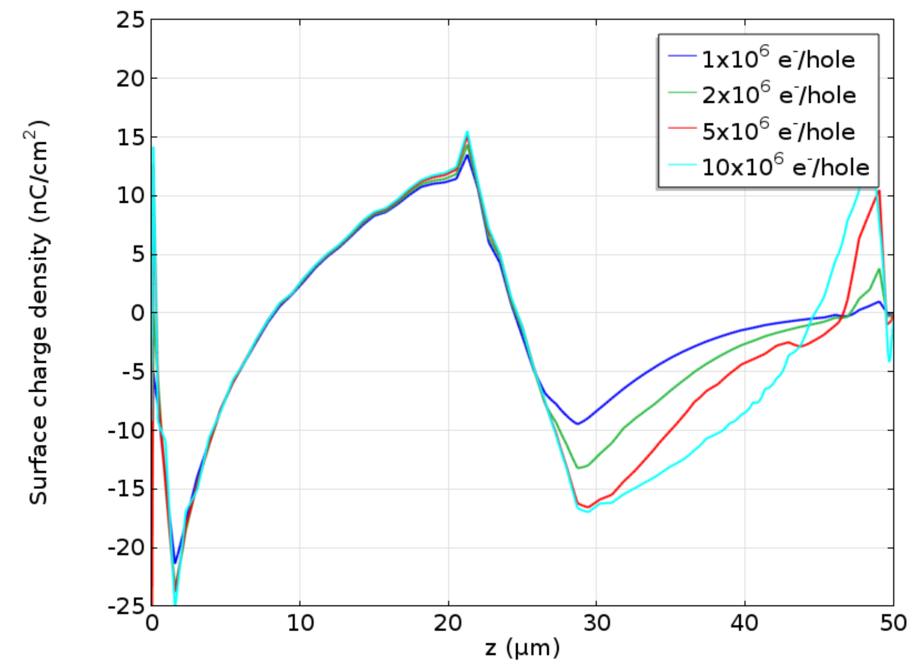
GEM charging up

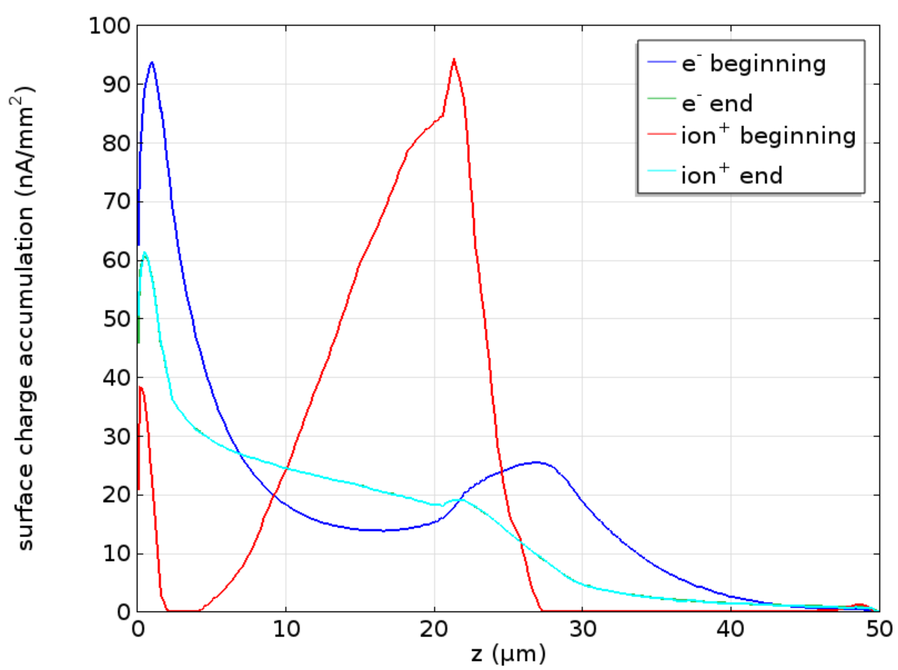


Single GEM, multiple stage GEM has ions from the bottom too. It is different…

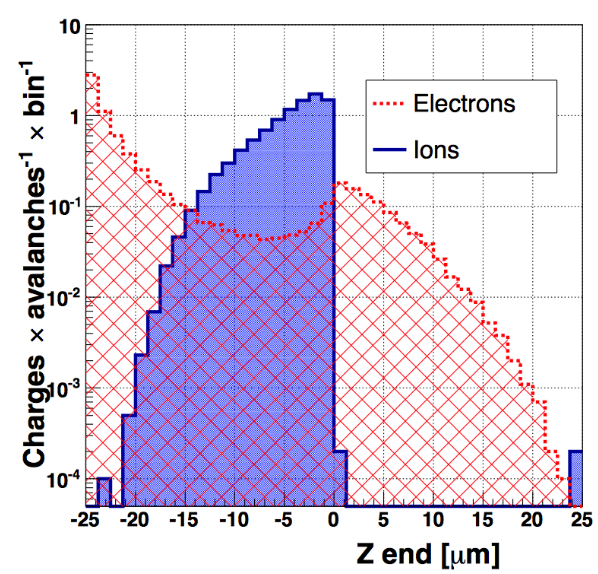
GEM charging up

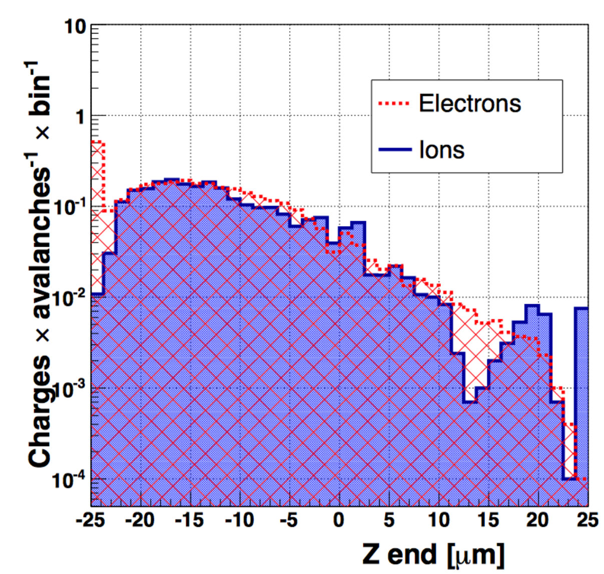




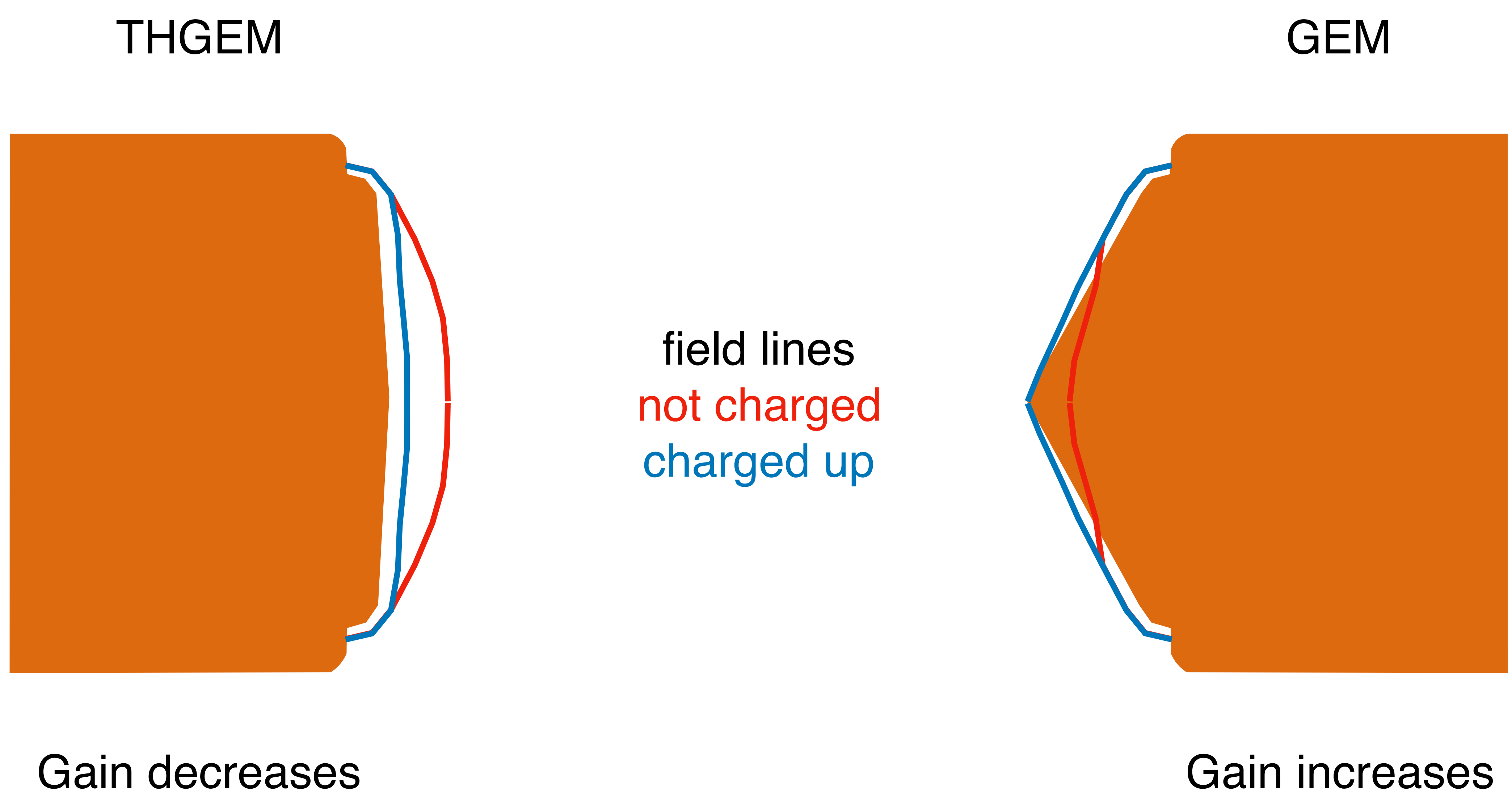


Correia *et al.*, JINST 9 (2014) P07025





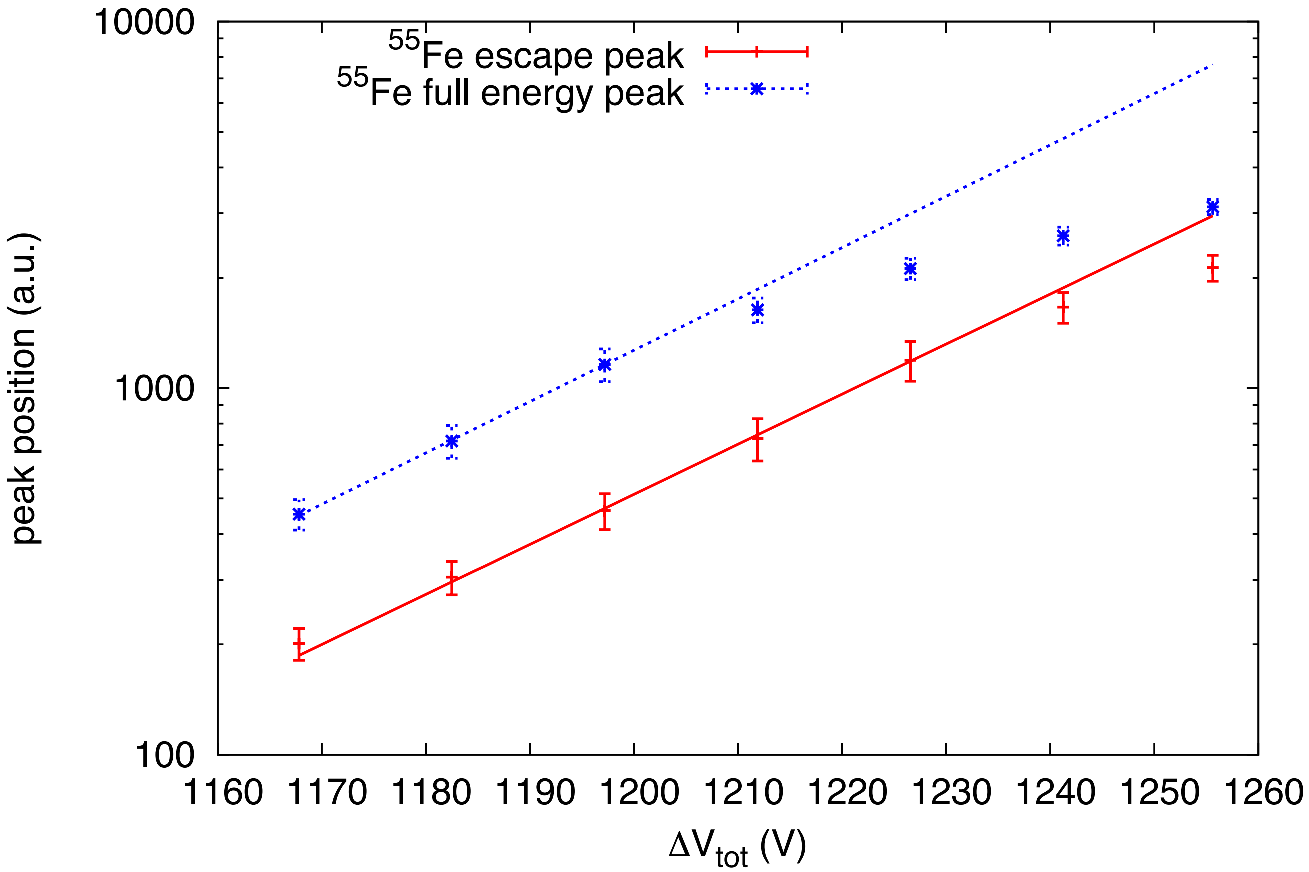
Charging up

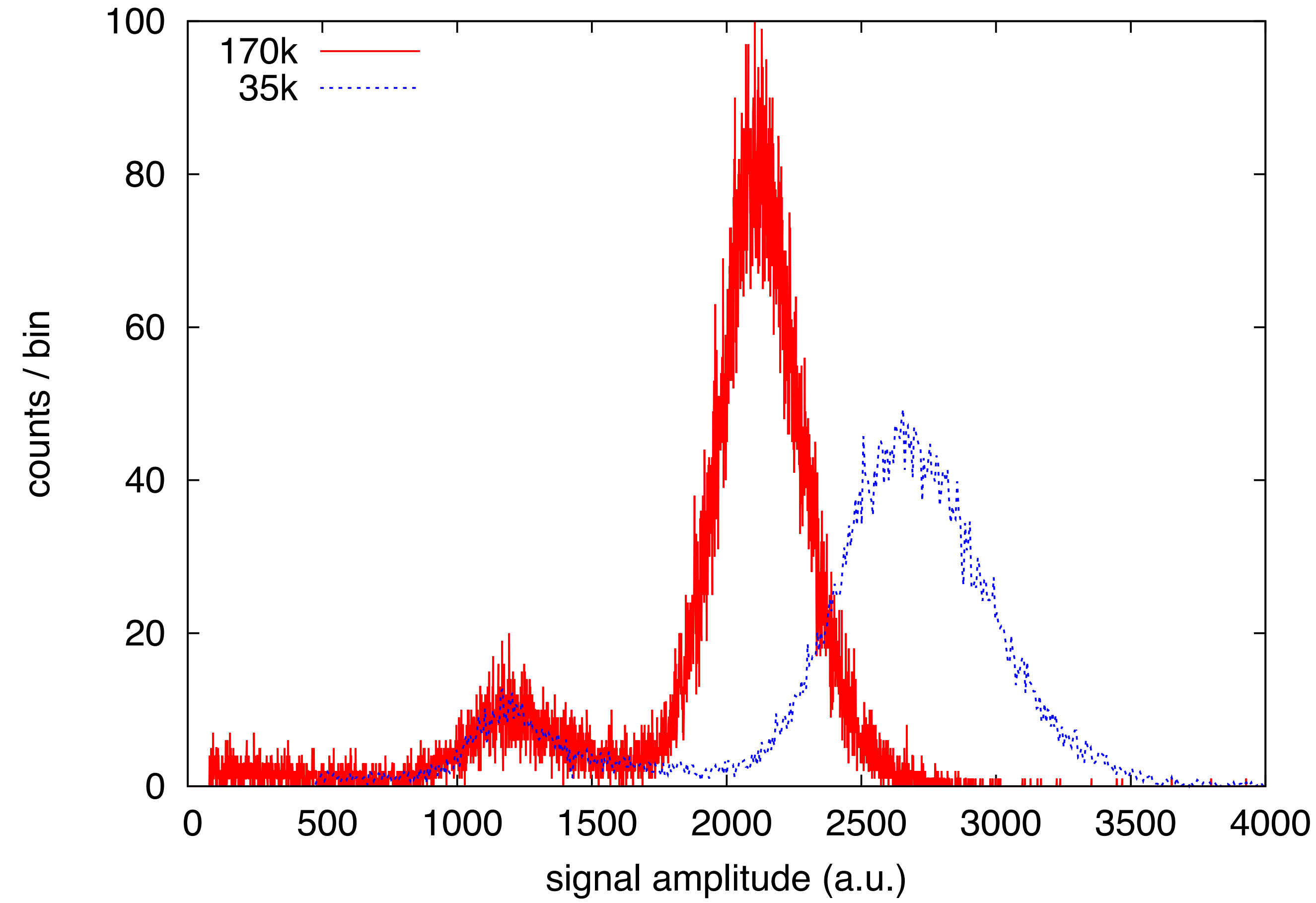

THGEM
GEM
field lines
not charged
charged up
Gain decreases
Gain increases

Example 6
Gain saturation

Measurement

Triple GEM data



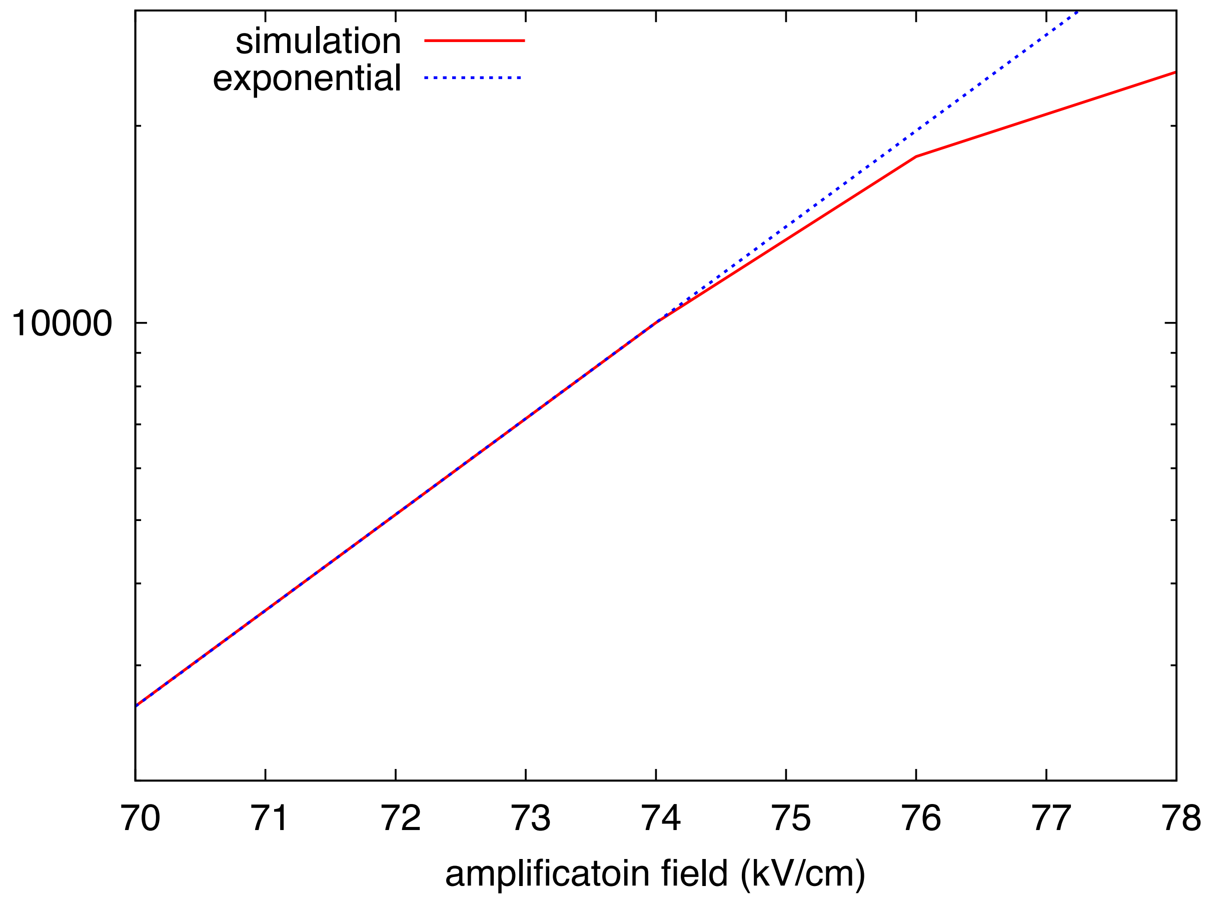


Saturation: deviation from the exponential
Saturation involves the full energy peak first
Prelude of a discharge

Each avalanche quenches its growth
Resolution improves:
large avalanches cannot grow,
small avalanches can still grow

Simulation

Simulation of a single avalanche in a GEM



Ions moving towards the entrance of the hole reduce the amplification field affecting the multiplication of the forthcoming electrons

Within the same computation framework, the saturation is qualitatively reproduced

Absolute gain mismatch is related to e- diffusion in several holes, not included in the computation

There is a maximum achievable gain in simulation too…

Example 6
Discharge

Notice

This work was (more than) inspired by a simulation of streamer by Paulo Fonte

With one fundamental difference: formation and propagation of streamers rely on electron diffusion only (no photo-ionisation)

Paulo Fonte computed the diffusion assisted streamer too (and before)

https://indico.cern.ch/event/89325/session/0/contribution/16/attachments/1089488/1554083/Calculation_of_streamer_development.pdf

Discharge

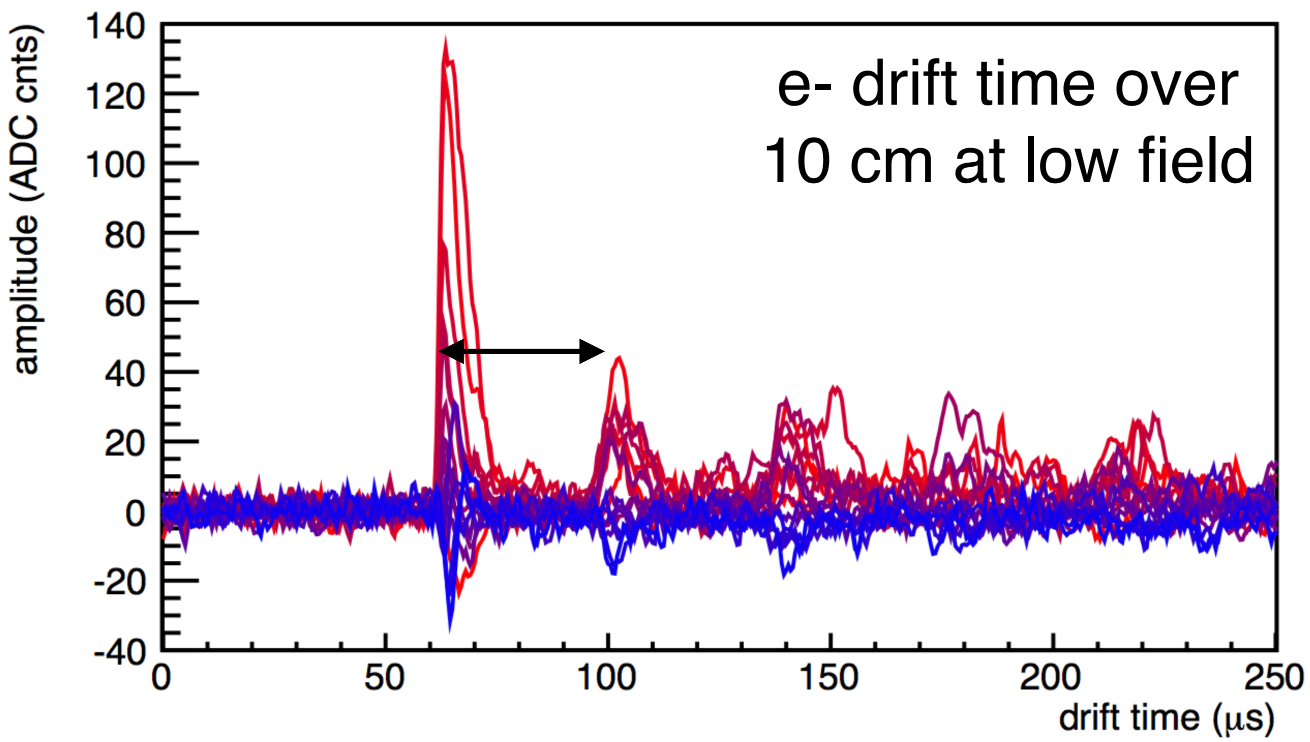
A generic term associated to a specific class of problems in gaseous detectors

Several kind of discharges:
Corona, Glow, Paschen, Arc, Streamer, …

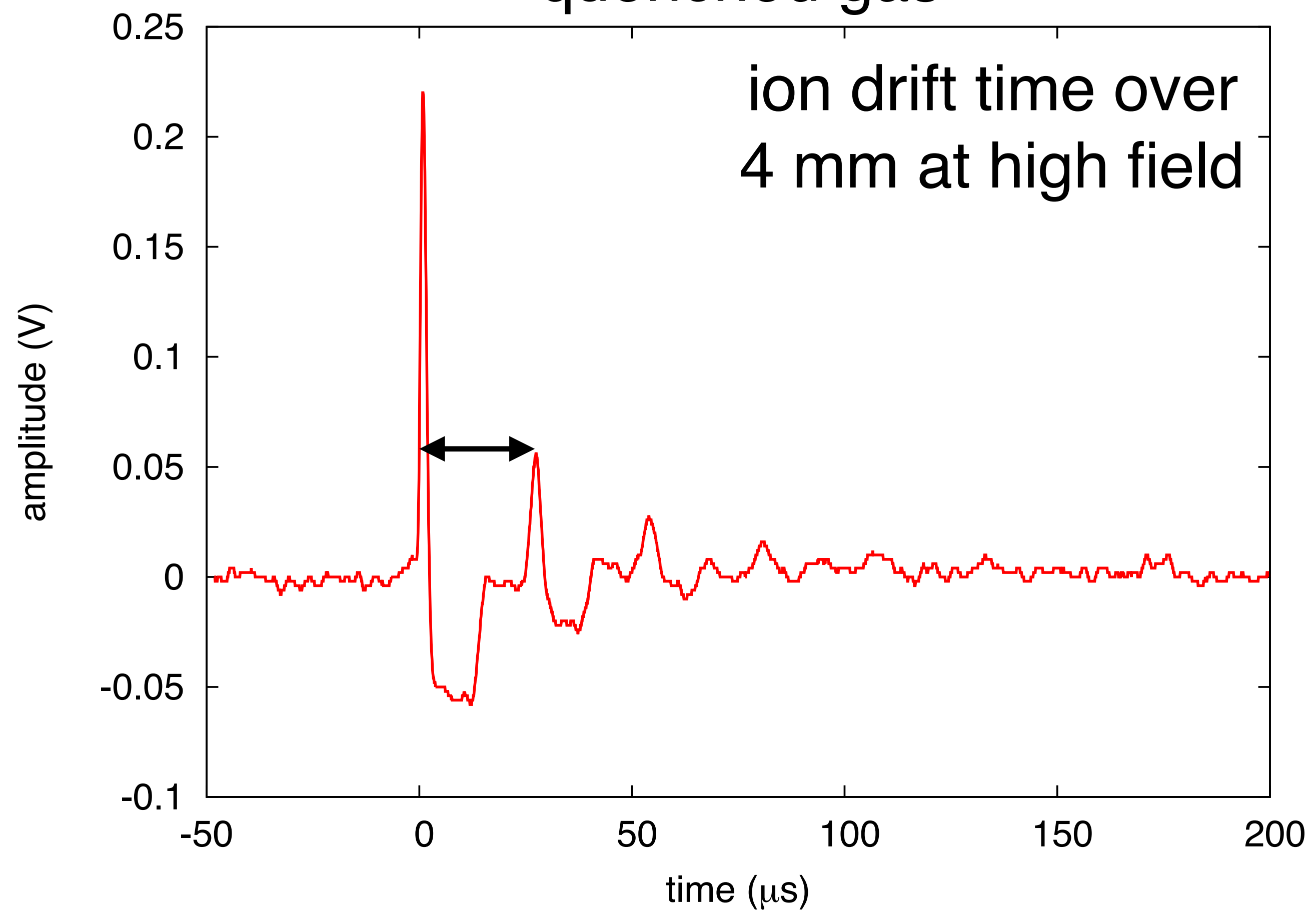
Each manifesting in specific situations and having distinct characteristics

À la Paschen

Photon feedback:
Exposed electrodes
in pure argon



Ion feedback:
Low pressure
quenched gas



Current diverges if next peak larger than the previous

Typically: event induced, increase of currents, **slow**

Streamer

Most relevant discharge (or discharge ignition) type for gaseous detectors in normal operation (personal opinion)

Sudden and **fast** (ns) evolution

Propagation also **towards** the **cathode**

Possibly with a precursor

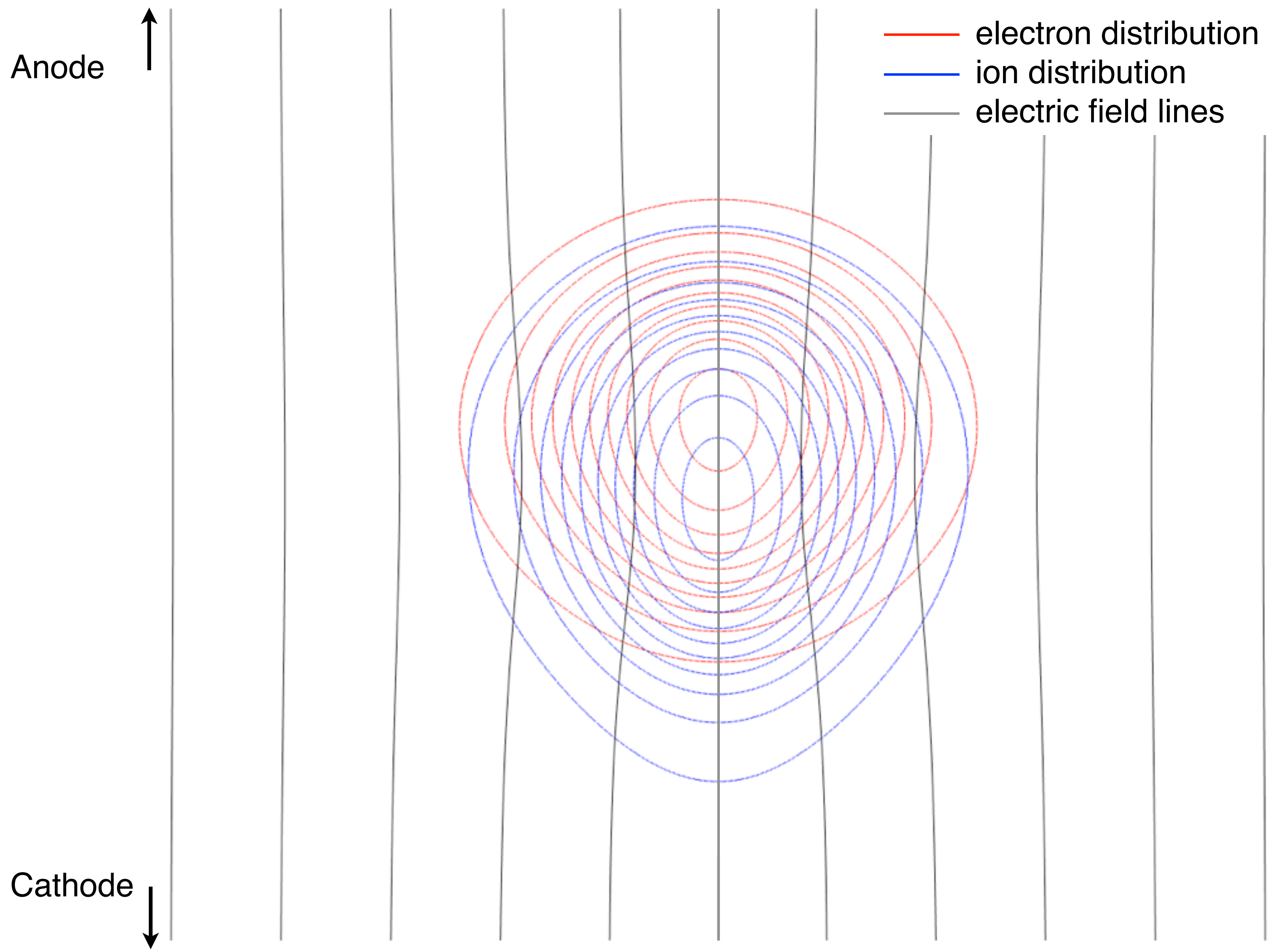
Streamer

Driven by electric field distortions due to large charge densities (Raether)

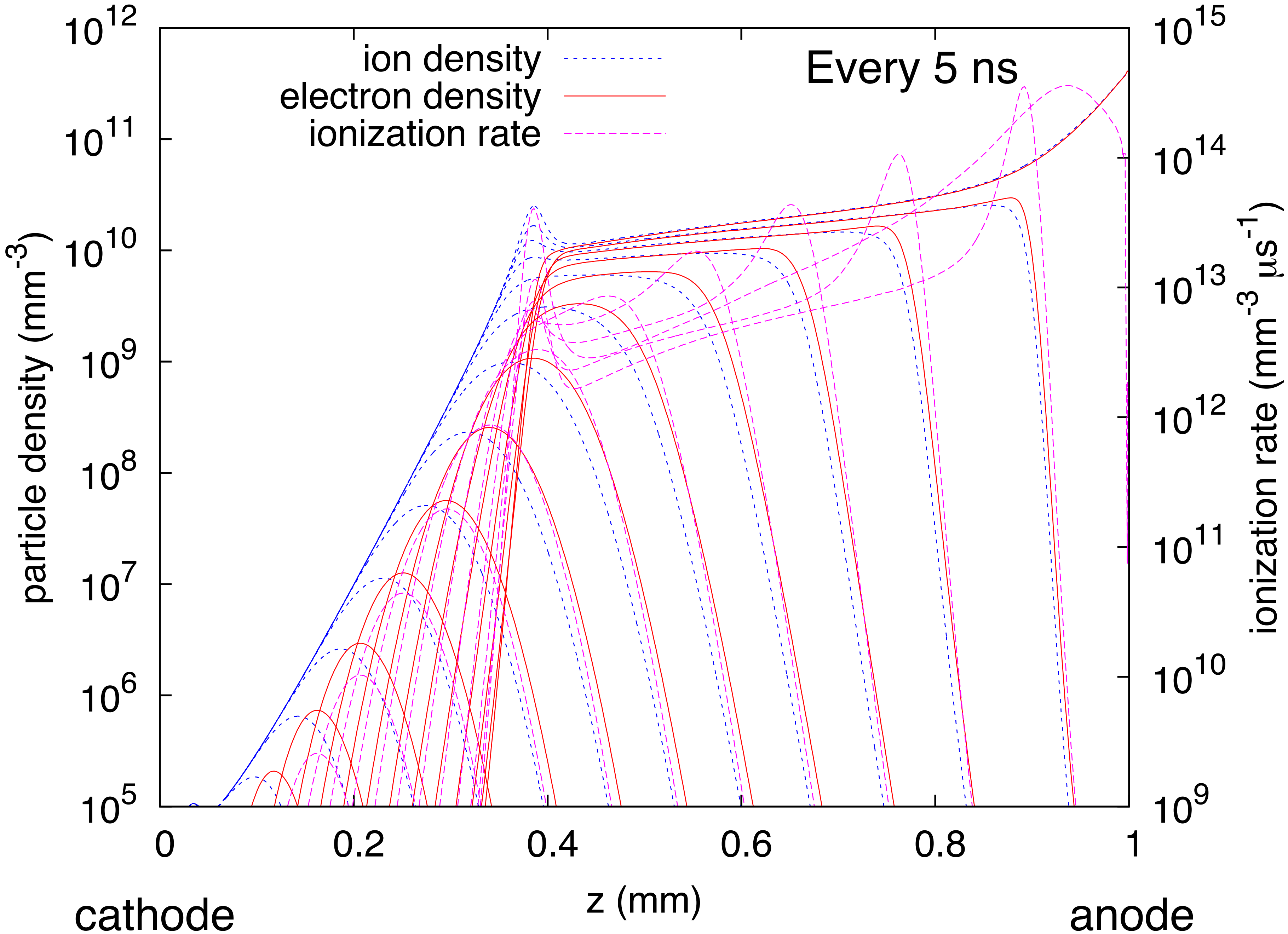
Rely on gas photo-ionisation and electron drift and diffusion

Hereafter photo-ionisation is neglected:
computation developed for pure argon, where scintillation photons are not able to ionise argon atoms

Basic mechanism

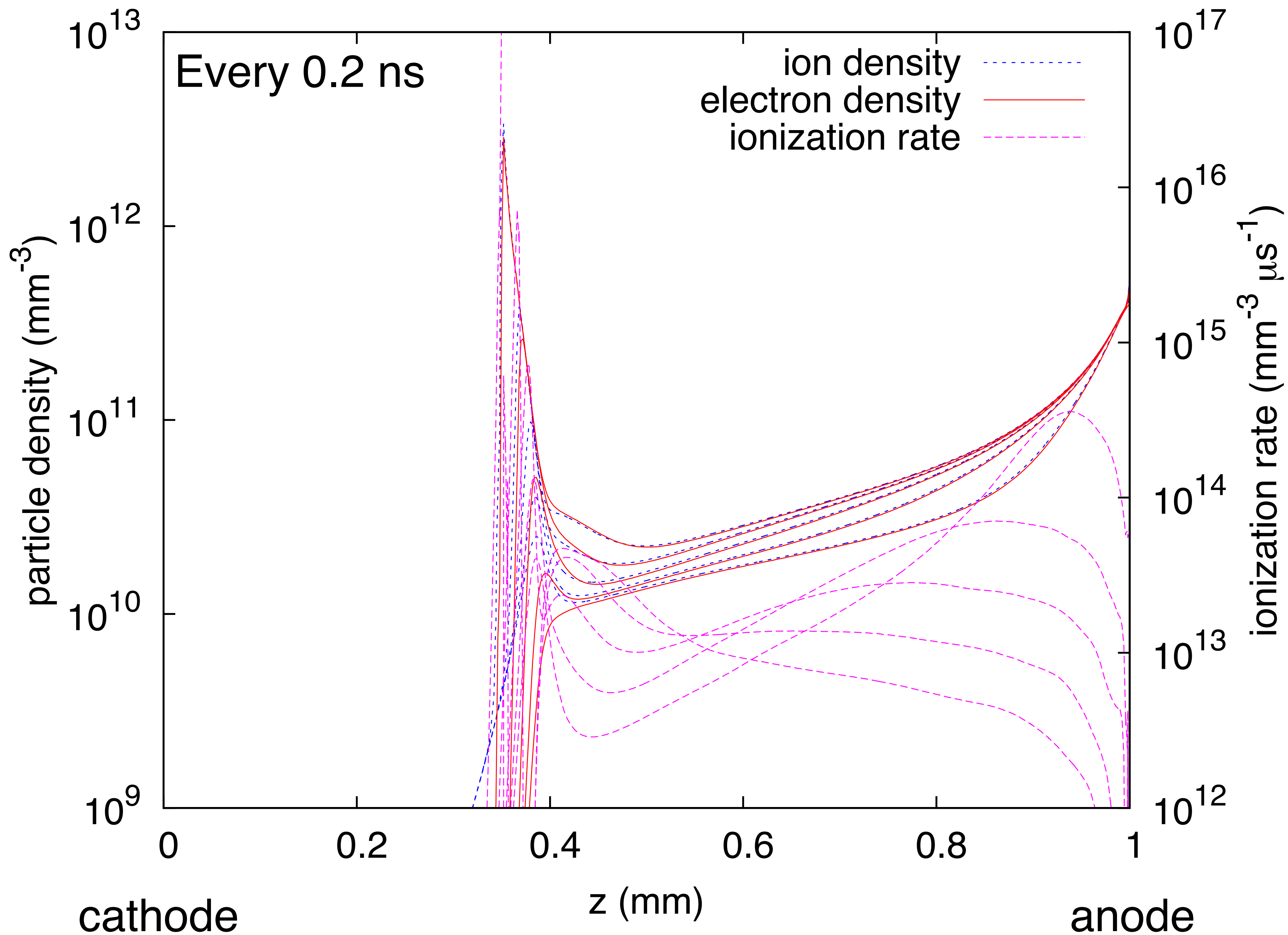


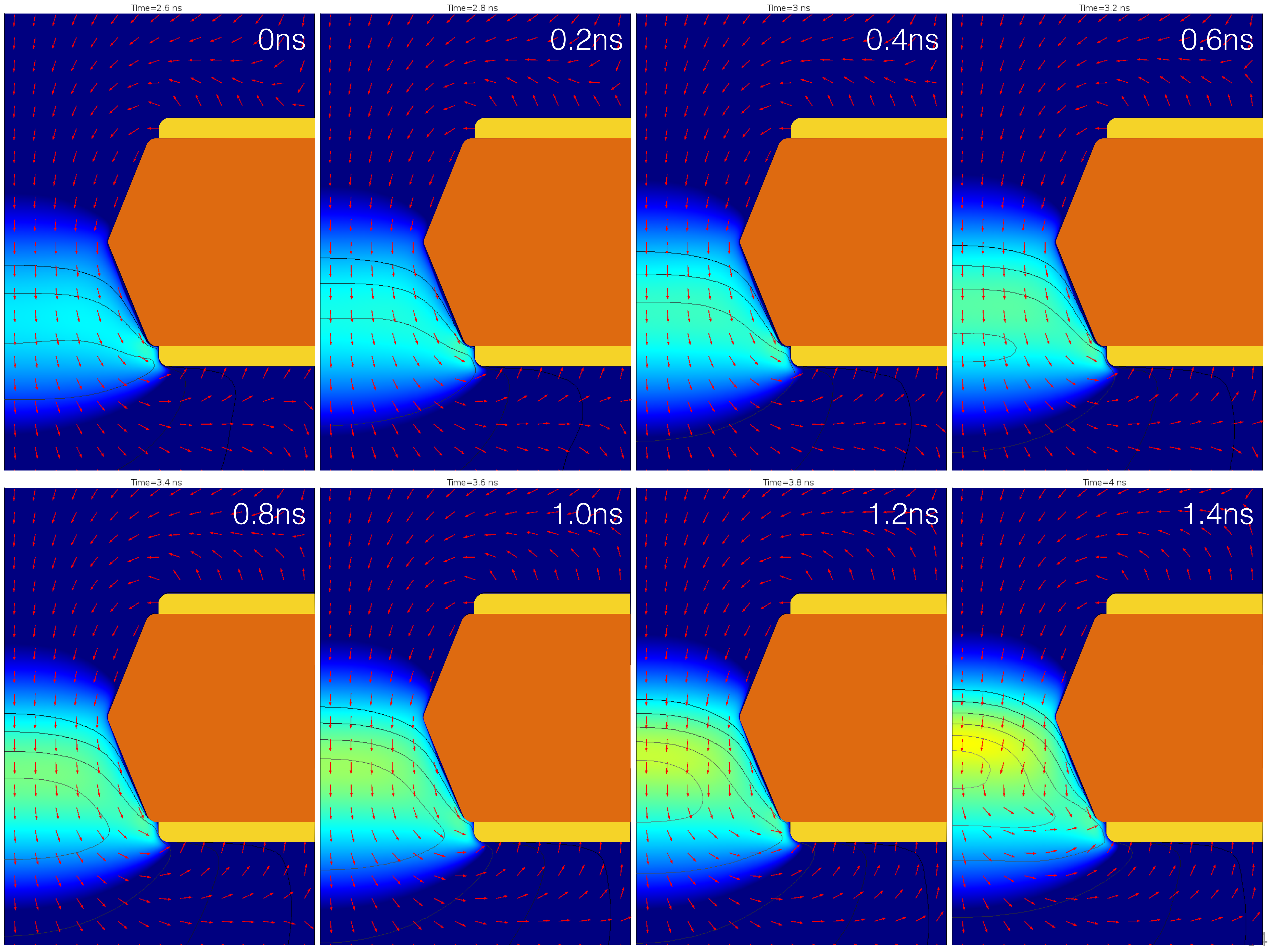
Evolution



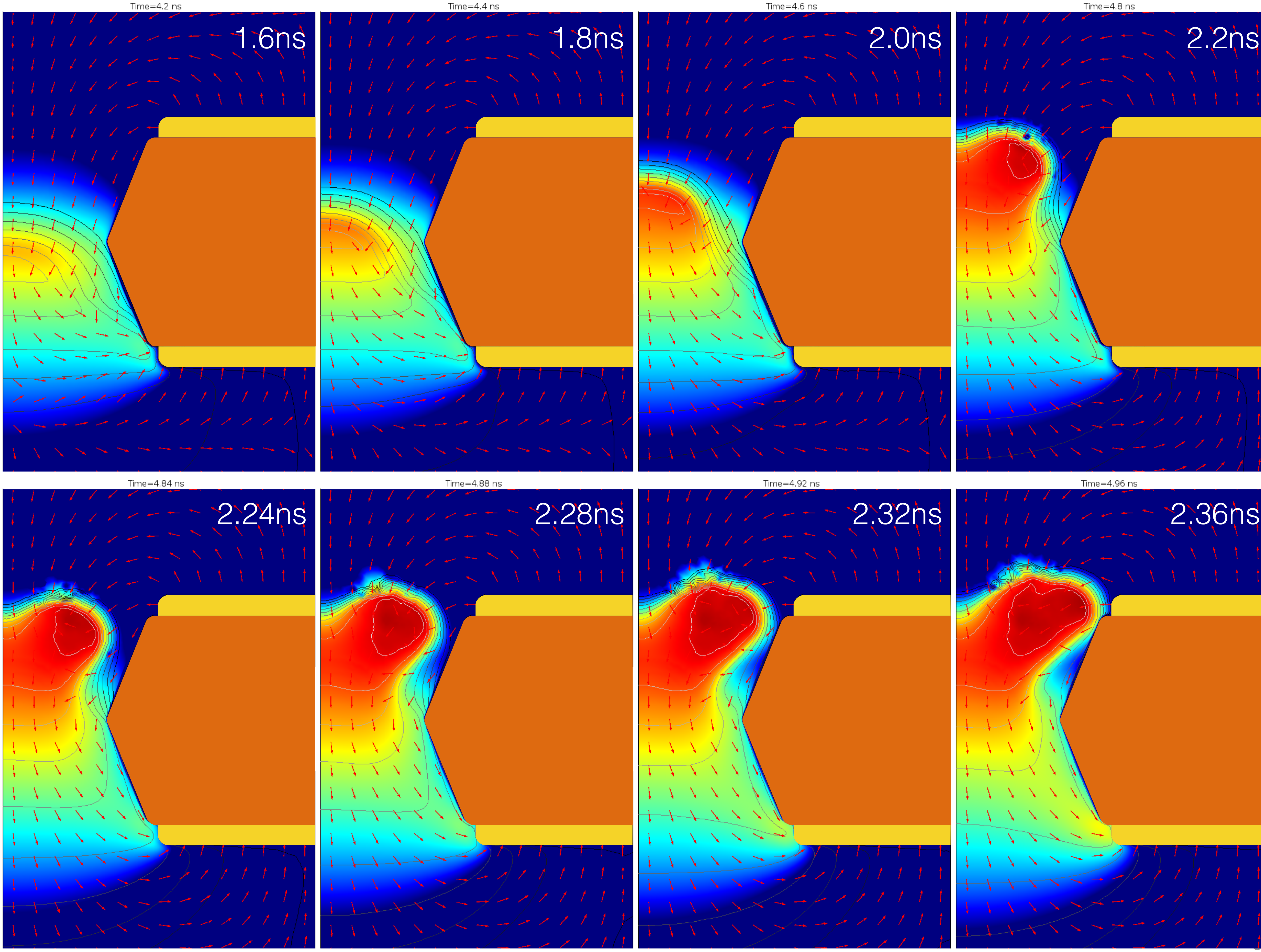
ion density
electron density
ionization rate
Every 5 ns
particle density (mm^{-3})
ionization rate (mm^{-3} μs^{-1})
z (mm)
cathode
anode
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
0
0.2
0.4
0.6
0.8
1

Evolution


Every 0.2 ns
ion density
electron density
ionization rate
particle density (mm^{-3})
ionization rate (mm^{-3} μs^{-1})
z (mm)
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
0
0.2
0.4
0.6
0.8
1
cathode
anode

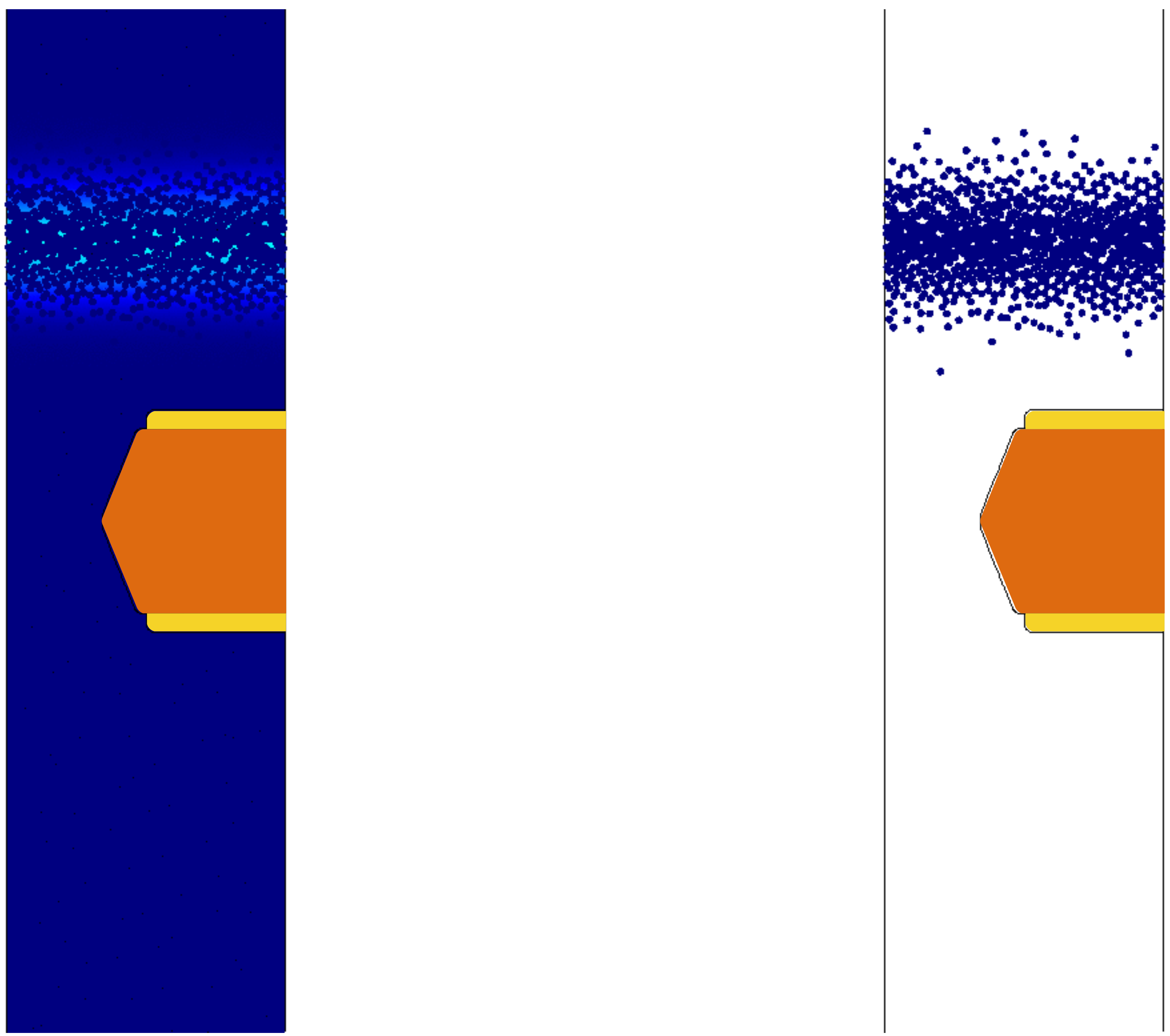


Time=2.6 ns
0ns
Time=2.8 ns
0.2ns
Time=3 ns
0.4ns
Time=3.2 ns
0.6ns
Time=3.4 ns
0.8ns
Time=3.6 ns
1.0ns
Time=3.8 ns
1.2ns
Time=4 ns
1.4ns


Time=4.2 ns
1.6ns
Time=4.4 ns
1.8ns
Time=4.6 ns
2.0ns
Time=4.8 ns
2.2ns
Time=4.84 ns
2.24ns
Time=4.88 ns
2.28ns
Time=4.92 ns
2.32ns
Time=4.96 ns
2.36ns

Two ideas

Particle tracking



Resistive electrodes

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J}$$

$$\vec{J} = \sigma \vec{E}$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \sigma \vec{E}$$

Too preliminary for showing more…

Time for discussion and live examples