On the Real and Apparent Positions of Moving Objects in Special Relativity: The Rockets-and-String and Pole-and-Barn Paradoxes Revisited and a New Paradox

J.H.Field

Département de Physique Nucléaire et Corpusculaire Université de Genève . 24, quai Ernest-Ansermet CH-1211 Genève 4.

E-mail: john.field@cern.ch

Abstract

The distinction between the real positions of moving objects in a single reference frame and the apparent positions of objects at rest in one inertial frame and viewed from another, as predicted by the space-time Lorentz Transformations, is discussed. It is found that in the Rockets-and-String paradox the string remains unstressed and does not break and that the pole in the Barn-and-Pole paradox never actually fits into the barn. The close relationship of the Lorentz-Fitzgerald Contraction and the relativity of simultaneity of Special Relativity is pointed out and an associated paradox, in which causality is apparently violated, is noted .

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1 Introduction

One of the most important new physical insights given in Einstein's seminal paper on Special Relativity (SR) [1] was the realisation that the Lorentz-Fitzgerald Contraction (LFC), which had previously been interpreted by Lorentz and Poincaré as an electro-dynamical effect, was most easily understood as a simple consequence of the space-time Lorentz Transformation (LT), i.e. as a geometrical effect [2].

It was pointed out 54 years later by Terrell [3] and Penrose [4] that when other important physical effects (light propagation time delays and optical aberration) are taken into account, as well as the LT, the moving sphere considered by Einstein in the 1905 SR paper would not appear to be flattened, in the direction of motion, into an ellipsoid, as suggested by Einstein, but rather would appear undistorted, but rotated. Shortly afterwards, Weinstein [5] pointed out that the LFC of a moving rod is apparent only if it is viewed in a direction strictly perpendicular to its direction of motion. It appears instead to be relatively elongated if moving towards the observer, and to be more contracted than the LFC effect if moving away from him. These effects are a consequence of light propagation time delays. A review [6] has discussed in some detail the combined effects of the LT, light propagation delays and optical aberration on the appearence of moving objects and clocks.

Another paper by the present author [7] pointed out that, in addition to the well-known LFC and Time Dilatation (TD) effects corresponding, respectively, to $t = constant$ and $x' = constant$ projections of the LT, two other apparent distortions of space-time may be considered: Time Contraction (TC) and Space Dilatation (SD) corresponding to the two remaining projections, $x = constant$ and $t' = constant$, respectively, of the LT [1].

The apparent nature of spatial distortions resulting from the LT is made evident by comparing the LFC with SD. In the former case the moving object appears shorter, in the latter longer, than the length as observed in its proper frame. Similarly, in TD, a moving clock appears to run slower than a similar clock at rest, while in TC identical moving clocks observed at a fixed position appear to be running faster than a similar, stationary, clock. A dynamical explanation has been proposed [8] for the LFC effect in the case of an extended object bound by electromagnetic forces, and of TD [9] in the case of various 'electromagnetic clocks'. No attempt has been made, to date, to find a dynamical explanation of the TC and SD effects. It is hard to see, in any case, how any 'dynamical' explanation can be given for the LFC of the proper distance between two isolated material objects in a common inertial frame, whereas, as first shown by Einstein, this is a simple space-time geometric consequence of the LT. As will be discussed below, one important reason for the misinterpretation of the 'Rockets-and String paradox' [10] is the incorrect assumption that extended objects undergo a 'dynamical' LFC that is different from that of the distance between two discrete objects.

The 'Rockets-and-String' and 'Pole-and-Barn' [11] as well as the similar 'Man-and-Grid' [12] and 'Rod-and-Hole' [13] paradoxes have all been extensively used in text

[1]The space and time coordinates: x, t; x', t' are measured in two inertial frames, S; S' in relative motion along their common x-axis

books on SR, for example in Taylor and Wheeler [14] and, more recently, by Tipler and Lewellyn [15]. In all cases the apparent LFC effects are correctly derived from the LT equations, but it seems to be nowhere realised that the 'real' positions and size of moving objects, in the sense that will be described below in Section 2, are different from the apparent ones derived from the LT. Indeed, the general assumption, in all of the papers and books just cited seems to be that there is no distinction between the real and apparent sizes and positions of moving objects, or, equivalently, that the LFC is a 'real' effect. The purpose of the present paper is to point out that this is not the case and that the 'real' and 'apparent' (i.e., those calculated using the LT) positions of moving objects are, indeed, distinct. The origin of this confusion between 'real' and 'apparent' positions and sizes of moving objects is not clear to the author. There is nothing in Einstein's 1905 SR paper to suggest that the LFC should be considered as a 'real' rather than an 'apparent' effect. Specifically, Einstein wrote [16]:

`Thus, whereas the X and Z dimensions of the sphere (and therefore every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened in the ratio` $1:\sqrt{1-v^2/c^2}$`, i.e. the greater the value of` v `the greater the shortening.`

In the original German the crucial phrase is: 'erscheint die X-Dimension im Verhältnis $1:\sqrt{1-v^2/c^2}$... '. The verb 'erscheinen', translated into English [17] means 'to appear'. Einstein never stated, in Reference [16], that the LFC is a 'real' effect.

The plan of this paper is as follows: The following Section discusses the real positions of objects as observed or measured in a single reference frame. Section 3 describes the apparent spatial positions of objects in uniform motion as predicted by the LT. The Rockets-and-String and Barn-and-Pole paradoxes are discussed in Sections 4 and 5 respectively. In Section 6, the close connection between the LFC and the relativity of simultaneity of SR is described and a new paradox of SR is pointed out. Section 7 contains a summary and conclusions.

2 The Real Positions of Moving Objects

The 'real' positions of one or more moving objects are defined here as those specified, for the different objects, in a single frame of reference. The latter may be either inertial, or with an arbitary accelerated motion. By introducing the concept of a co-moving inertial frame, at any instant of the accelerated motion, the distance between the objects can always be defined as the proper distance between them in a certain inertial frame. If this distance in the different co-moving inertial frames is constant the 'real' distance is said to be constant. No distinction is made between the real distance between the points on a rigid extended object and that between discrete, physically separated, objects coincident in space-time with these points. This is because the LT, that relates only space-time events in different inertial frames, treats, in an identical manner, points on extended or discrete physical objects.

The utility of the science of the 'real' positions of moving objects (astronomy, railways,

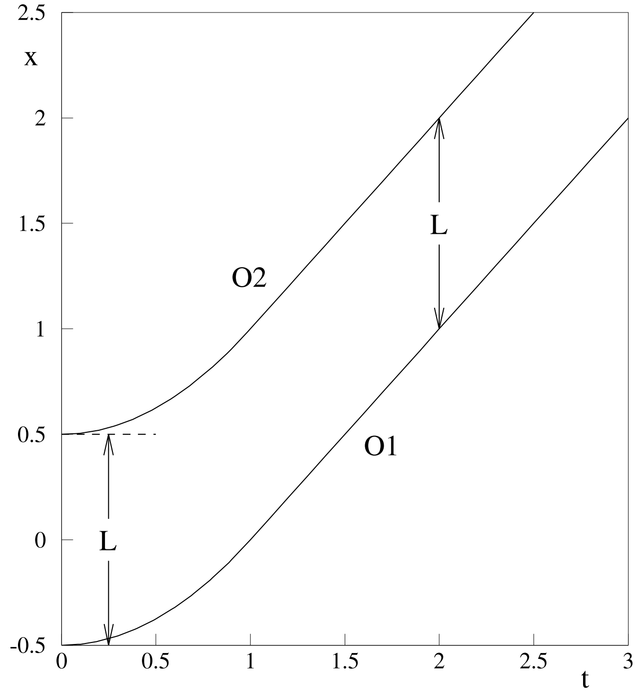

Figure 1: *Space-time trajectories in the frame S of the real positions of O1 and O2 when subjected to constant and identical proper accelerations. Units are chosen with* $c = 1$. *Also* $L = a = 1$, $t_{acc} = \sqrt{3}$.

military ballistics, air traffic control, space travel, GPS satellites...) is evident and the validity of the basic physical concepts introduced by Galileo (distance, time, velocity and acceleration) are not affected, in any way, by SR. The 'real' positions of objects in a given reference frame are those which must be known to, for example, avoid collisions between moving objects in the case of railway networks, or, on the contrary, to assure them in the case of military ballistics or space-travel. From now on, in this paper, the words 'real' and 'apparent' will be written without quotation marks.

For clarity, a definite measuring procedure to establish the real distance between two moving objects in a given frame of observation is introduced. This will be useful later when discussing the Barn-and-Pole paradox. Suppose that the objects considered move along the positive x-axis of an inertial coordinate system S. The co-moving inertial frame of the objects is denoted by S'. It is imagined that two parallel light beams cross the x-axis in S, at right angles, at a distance ℓ apart. Each light beam is viewed by a photo-cell and the moving objects are equipped with small opaque screens that block the light beams during the passage of the objects. Two objects, O1 and O2, with x(O2) > x(O1), moving with the same uniform velocity, v, will then interrupt, in turn, each of the light beams. Suppose that the photo-cells in the forward (F) and backward (B) beams are equipped with clocks that measure the times of extinction of the beams to be (in an obvious notation) $t(F1)$, $t(F2)$, $t(B1)$ and $t(B2)$[2]. Consideration of the motion of the

[2]'forward' and 'backward' are defined from the viewpoint of the moving objects. Thus the forward beam lies nearest to the origin of the x-axis

objects past the beams gives the following equations;

$$
\begin{aligned}
t(B1) - t(F1) &= \frac{\ell}{v} && (2.1) \\
t(B2) - t(F2) &= \frac{\ell}{v} && (2.2) \\
t(B2) - t(F1) &= \frac{\ell - L}{v} && (2.3)
\end{aligned}
$$

where L is the real distance between the moving objects. Eqns(2.1) and (2.2) give the times of passage of the objects O1 and O2, respectively, between the light beams, whereas Eqn(2.3) is obtained by noting that, if $\ell > L$, each object moves the distance $\ell - L$ during the interval $t(B2) - t(F1)$. If $\ell < L$ each object moves a distance $L - \ell$ during the interval $t(F1) - t(B2)$ and the same equation is obtained. Taking the ratios of Eqn(2.3) to either Eqn(2.1) or Eqn(2.2) yields, after some simple algebra, the relations:

$$
\begin{aligned}
L &= \ell \frac{[t(B1) - t(B2)]}{t(B1) - t(F1)} && (2.4) \\
L &= \ell \frac{[t(F1) - t(F2)]}{t(B2) - t(F2)} && (2.5)
\end{aligned}
$$

Subtracting Eqn(2.3) from Eqs(2.1) or (2.2), respectively, gives:

$$
\begin{aligned}
t(B1) - t(B2) &= \frac{L}{v} && (2.6) \\
t(F1) - t(F2) &= \frac{L}{v} && (2.7)
\end{aligned}
$$

Taking the ratios of Eqn(2.6) to (2.2) and Eqn(2.7) to (2.1) then yields two further equations, similar to (2.4) and (2.5) above:

$$
\begin{aligned}
L &= \ell \frac{[t(B1) - t(B2)]}{t(B2) - t(F2)} && (2.8) \\
L &= \ell \frac{[t(F1) - t(F2)]}{t(B1) - t(F1)} && (2.9)
\end{aligned}
$$

Equations (2.4),(2.5),(2.8) and (2.9) show that any three of the four time measurements are sufficient to determine the real separation, L, between the two moving objects. In these equations the times: $t(F2)$, $t(B1)$, $t(F1)$ and $t(B2)$, respectively, are not used to determine L. In order to combine all four time measurements to obtain the best, unbiased, determinations of v and L, Eqns(2.1) and (2.2) may be added to obtain:

$$
t(B1) + t(B2) - t(F1) - t(F2) = \frac{2\ell}{v} \qquad (2.10)
$$

while subtracting two times Eqn(2.3) from (2.10) gives:

$$
t(B1) - t(B2) + t(F1) - t(F2) = \frac{2L}{v} \qquad (2.11)
$$

The velocity, v, is obtained by transposing Eqn(2.10):

$$
v = \frac{2\ell}{t(B1) + t(B2) - t(F1) - t(F2)} \qquad (2.12)
$$

while the ratio of Eqn(2.11) to (2.10) gives:

$$L = \ell \frac{[t(B1) - t(B2) + t(F1) - t(F2)]}{t(B1) + t(B2) - t(F1) - t(F2)} \tag{2.13}$$

It is interesting to note that the real spatial positions, as well as the instantaneous velocity and acceleration, at any time, of two objects subjected to a symmetric, uniform, acceleration in the frame S, can also be determined from the four time measurements just considered. In this case there is no redundancy, the time measurements determine four equations which may be solved for the four quantites just mentioned. In the case of uniform motion, the constant velocity hypothesis may be checked by comparing the independent determinations of v provided by Eqns(2.1) and (2.2). Furthermore, as already mentioned, any three of the four time measurements is sufficient to determine L. Evidently, if $t(F1) = t(B2)$, then $L = \ell$ in the case of an arbitary accelerated motion of the two objects. Thus, by varying ℓ, the real distance between the co-moving objects can be determined even if the acceleration program of their co-moving frame is not known.

The two objects, moving with equal and constant velocities along the x-axis in S, discussed above, are now considered to be set in motion by applying identical acceleration programs to two objects initially at rest and lying along the x-axis in S. The two objects considered are then, by definition, subjected to the same acceleration program in their common rest frame, or, what is the same thing, their common rest frame, (with respect to which the two objects are, at all times, at rest) is accelerated. Under these circumstances the distance between the objects remains constant in the instantaneous co-moving inertial frame of the objects. At the start of the acceleration procedure, the instantaneous co-moving inertial frame is S, at the end of the acceleration procedure it is S'. Therefore the separation of the objects in S at the start of the acceleration procedure is the same as that in S' at the end of it. Note that there is no distinction between the real and apparent distances for objects at rest in the same inertial frame. Also 'relativity of simultaneity' can play no role, since the proper time of both objects is always referred to the same co-moving inertial frame. Since the acceleration program of both objects starts at the same time in S, and both objects execute identical space time trajectories, the real separation of the objects must also remain constant. This necessarily follows from space-time geometry. Similarly, since the acceleration program stops at the same time in S' for both objects the real separation of the objects remains constant in this frame and equal to the original separation of the objects in S. This behaviour occurs for any symmetric acceleration program, and is shown, for the special case of a constant acceleration in the rest frame of the objects (to be calculated in detail below), in Fig 1. Since, in the above discussion, both objects are always referred to the same inertial frame there is no way that SR can enter into the discussion and change any of the above conclusions. Indeed, SR is necessary to derive the correct form of the separate space-time trajectories in S, but the symmetry properties that guarantee the equalites of the real separations of the objects cannot be affected, in any way, by SR effects.

Two objects, O1 and O2, originally lying at rest along the x-axis in S and separated by a distance L are now simultaneously accelerated, during a fixed time period, t_{acc}, in S, starting at $t = 0$, with constant acceleration, a, in their common proper frame, up to a relativistic velocity $v/c \equiv \beta = \sqrt{3}/2$, corresponding to $\gamma = 1/\sqrt{1-\beta^2} = 2$, and

$t_{acc} = c\sqrt{3}/a$. The equations giving the velocity v and the position x in a fixed inertial frame, using such an acceleration program, were derived by Marder [18] and more recently by Nikolic [19] and Rindler [20]. The positions and velocities of the objects in the frame S are:

for $t \leq 0$

$$\begin{aligned} v_1 &= v_2 = 0 &(2.14)\\ x_1 &= -\frac{L}{2} &(2.15)\\ x_2 &= \frac{L}{2} &(2.16) \end{aligned}$$

for $0 < t < t_{acc}$

$$\begin{aligned} v_1(t) &= v_2(t) = v(t) = \frac{act}{\sqrt{c^2+a^2t^2}} &(2.17)\\ x_1(t) &= c\left[\frac{\sqrt{c^2+a^2t^2}-c}{a}\right] - \frac{L}{2} &(2.18)\\ x_2(t) &= c\left[\frac{\sqrt{c^2+a^2t^2}-c}{a}\right] + \frac{L}{2} &(2.19) \end{aligned}$$

and for $t \geq t_{acc}$

$$\begin{aligned} v_1(t) &= v_2(t) = v(t_{acc}) &(2.20)\\ x_1(t) &= v(t_{acc})(t-t_{acc}) + x_1(t_{acc}) &(2.21)\\ x_2(t) &= v(t_{acc})(t-t_{acc}) + x_2(t_{acc}) &(2.22) \end{aligned}$$

The origins of S and S' have been chosen to coincide at $t = t' = 0$. The real positions of the objects in S are shown, as a function of t for $a = 1$ and $t_{acc} = \sqrt{3}$, in Fig.1. The velocities of the two objects are equal at all times, as is also the real separation of the objects $x_2 - x_1 = L$. SR is used only to derive Eqn(2.17). The time-varying velocity is then integrated according to the usual rules of classical dynamics in order to obtain Eqns(2.18),(2.19) for the positions of the objects during acceleration. These are, by definition, the *real* positions of the objects O1 and O2 in S. The equalities of the velocities and the constant real separations are a direct consequence of the assumed initial conditions and the similarity of the proper frame accelerations of the objects. These are the sets of equations that must be used to specify the distance between O1 and O2 and any other objects, whose real positions are specified in S, in order to predict collisions or other space-time interactions of the objects.

3 The Apparent Positions of Moving Objects in Special Relativity

In order to account correctly for the actual appearence of moving objects, not only the LT but also the effects of light propagation delays to the observer, and optical aberration must be properly taken into account. In the following, it is supposed that the necessary corrections for the two latter effects have been made, so that only consequences of the LT are discussed. In this case the apparent position x^A in a 'stationary' inertial frame, S, at time[3], t, of an event at x', t' in the inertial frame, S', moving with velocity v along the positive x-axis in S, is given by the space-time LT:

$$x^A = \gamma(x' + vt') \quad (3.1)$$

$$t = \gamma(t' + \frac{vx'}{c^2}) \quad (3.2)$$

In Eqns(3.1) and (3.2), clocks in S and S' synchronised so as to record the same time at $t = t' = 0$ when $x^A = x' = 0$.

Since 'observation' implies the interaction, with a detection apparatus, of one or more photons, originating from the observed object, it is convenient to define the space-time events that are related by the LT to correspond, in S', to the points of emission (or reflection) of a photon from a source at rest, and in S to the space-time point at which the emission or reflection of this photon is observed. It is assumed that, after the acceleration procedure described in the previous Section, the frame S' moves with constant velocity $v = V$ along the x-axis of S. How the object will appear when viewed from S, depends on which photons, scattered from or emitted by the object, are selected by the observer. In the case of the LFC discussed by Einstein [1] the observer in S requires that the photons emitted from O1 and O2 are observed simultaneously in S (the $\Delta t = 0$ projection of the LT). As is easily derived from Eqns(3.1) and (3.2), the separation of the two objects then appears to be L/γ where L is their real separation in S'. Another simple possibility [7] is to require that the two objects are both illuminated for a very short period of time in their proper frame, yielding a 'transient luminous object'. In this case, a quite complicated series of events is observed in S. A line image (whose width depends on the period of illumination) is seen to move with velocity c/β in the same direction as that of the moving object, sweeping out a total length γL. This is the so-called 'Space Dilatation' (SD) effect (the $\Delta t' = 0$ projection of the LT) [7]. As discussed in more detail below, both the LFC and TD effects are direct consequences of the 'relativity of simultaneity' of SR introduced by Einstein. In both LFC and TD, the space-time events in S' are associated with the real positions of the objects, as defined above, in this frame. However, since it is clear that the apparent positions of these space-time events, as observed in S, depend upon the mode of observation, being different for the LFC and TD, they evidently cannot be identified with the real positions of the objects in this frame. The relation of the real and apparent positions will now be discussed in detail for the two 'paradoxes' of SR mentioned above.

4 The Rockets-and-String Paradox

The idea that the LFC could induce mechanical stress in a moving extended body was introduced by Dewan and Beran [10], criticised by Nawrocki [21] and defended by Dewan [11] and Romain [22]. Dewan and Beran discussed symmetric acceleration of two rockets, originally at rest in the frame S, in a way very similar to the acceleration of the

[3]Notice that there is no possible distinction between the 'time' and the 'apparent time' in the frame S. t is simply the time recorded by a synchronised local clock in S.

two objects O1 and O2 described in Section 2 above, except that no specific acceleration program was defined. It was correctly concluded that the real distance between the objects in S would remain unchanged throughout the aceleration procedure. However, it was not stated that, after acceleration, the proper separation between the rockets is the same as their original separation in S. Dewan and Beran then introduced a continous string attached between the rockets during the acceleration and drew a distinction between two distances:

`(a) the distance between two ends of a connected rod and (b) the distance between two objects which are not connected but each of which independently and simultaneously moves with the same velocity with respect to an inertial frame`

It was then stated (without any supporting argument) that the distance (a) is subject to the LFC and (b) not. Replacing the 'connected rod' of (a) by a continous string attached between the rockets, it was concluded that the string would be stressed and ultimately break, since the distance between the rockets does not change, whereas the string shrinks due to the LFC. Several correct arguments were given why the *real* distance between the rockets does not change. The incorrect conclusion that the string would be stressed and break was due to the failure to discriminate between the *real* separation of the rockets (correctly calculated) and the *apparent* contraction due to the LFC, which as correctly pointed out by Nawrocki [21] applies equally to the distance between the ends of the string and that between the points on the rockets to which it is attached. Dewan and Beran's distinction between the distances (a) and (b) is then wrong. Both the distance between the points of attachment of the string and the length of the string undergo the same apparent LFC. There is no stress in the string. It does not break.

Dewan did not respond to Nawrocki's objection that (correctly) stated the equivalence of the length of an extended object and the distance between two independant objects separated by a distance equal to this length, but instead introduced a new argument, claiming that Nawrocki had not correctly taken into account the relativity of simultaneity of SR. Dewan considered the sequence of events corresponding to the firing of the rockets as observed in S', the co-moving frame of the rockets after acceleration. It was concluded that the spring breaks because, in this case, the final separation of the rockets is γL (this is just the SD effect described above). But for this, the LT must be applied to space time events on the rockets (and not to the string), whereas when considering an observer in S, Dewan and Beran had applied the LT to the string (and not to the rockets)! Viewed from S' both the distance between the rockets and the length of the string undergo the apparent SD effect, there is no stress in the string and it does not break.

All of the confusion and wrong conclusions in References [10] and [11] result, firstly, from the failure to notice the strict equivalence of separations of type (a) and (b) and secondly from not realising that the apparent and real separations of the moving rockets are not the same. Even Nawrocki, who correctly pointed out the fallacy of assigning the LFC to separations of type (a) but not to type (b) seems not to have realised the correctness of Dewan and Beran's calculation of the real distance between the rockets. He correctly stated that the apparent contractions of the length of the string and the distance between the rockets would be the same, but did not discriminate this apparent distance from the real separation of the rockets in the frame S.

The acceleration program discussed in Section 2 above (and indeed any programme which is an arbitary function of proper time) when applied in a symmetrical manner to the two objects O1 and O2, will leave unchanged the proper separation of the objects in their co-moving frame. On the other hand. the 'instantaneous' acceleration considered by Dewan apparently resulted in a change in the proper separation of the rockets. If Dewan had considered instead a physically realistic acceleration program, such as that discussed in Section 2 above (which must necessarily occupy a finite period of time) no change in the proper separation of the rockets in their co-moving frame would have been predicted. This is shown in Fig.1b above. In fact, Dewan did not consider the whole acceleration process, but instead calculated the apparent positions in S', according to the LT, of the space-time events corresponding to the firing of the rockets in S, i.e. the start of the acceleration programme. These apparent positions are not the same as the real positions of the rockets in S'. The apparent length of a rod uniformly accelerated in its proper co-moving frame has recently been calculated by Nikolic [19]. It was correctly noted in this paper, contrary to the conclusion of Dewan, that the real length of the rod remains unchanged in this frame.

In a comment on Reference [10], Evett and Wagness [23] stated that the distance between the rockets, given by Dewan and Beran as that between the tail of the first rocket and the head of the second, would not remain constant in S, due to the LFC of the rockets, whereas the distance between any two corresponding points would, Thus, like Dewan and Beran, Evett and Wagness assumed that extended material objects (rockets, strings) undergo the LFC, but not the spatial separation of the rockets in their co-moving frame. Thus Evett and Wagness also stated that the string will break. Actually, the real lengths of the rockets as well as the distance between them, in the frame S, remain unchanged during acceleration, but, as correctly stated by Nawrocki, they both undergo the apparent LFC.

In Section 3 of Reference [11] Dewan considered an observer at rest in the co-moving frame of the accelerated rocket, R2, with the smallest x-coordinate. He concluded that, viewed from this frame, the second rocket, R1, would have a non-vanishing velocity in the x-direction that stresses the string and causes it to break. Such behaviour is clearly in contradiction with the assumed symmetry of the acceleration program. At any instant the two rockets have a common co-moving frame within which their relative velocity is zero. Dewan's argument was later supported by Romain [22] who considered a constant acceleration of the rockets in their co-moving proper frame as in Eqns.(2.17)-(2.19) above. This conclusion however was founded on a misinterpretation of the space-time diagram (Fig.1 of Reference [22]) describing the motion of the rockets. In reference to this figure it was incorrectly stated that 'B_2B_1' and A_2A_1 represent the same "proper length" '. In fact, the proper separation of the rockets in the frame S' (with space and time coordinates X'^1 and X'^4) corresponding to the space-time point B_2, on the trajectory of R2, in the frame S (with space and time coordinates X^1 and X^4) is represented by the distance between B_2 and the intersection of the X'^1 axis with the tangent to the trajectory of R1 at B_1 (say, the point B_1^t) not B_2B_1'. The latter is the distance between B_2 and the intersection of the X'^1 axis with the trajectory of R1. The apparent distance in S corresponding to the segment $B_2B_1^t$ (a $\Delta X'^4 = 0$ projection) is found to be γB_2B_1 so that, from the SD effect described in Section 3 above, the proper separation of the rockets in S' is $B_2B_1 = A_2A_1$, i.e. the same as their initial separation in S. Thus it is the segment $B_2B_1^t$, not B_2B_1', that represents the same proper length as A_2A_1. Since the velocity in S of R1 at B_1' is greater than that of R_2 at B_2, Romain was lead to the same erroneous conclusion as Dewan, that the proper distance between the rockets is increasing, causing the string to break. It may be noted that this conclusion is actually in contradiction with Romain's own (but incorrect) statement that A_2A_1 and B_2B_1' represent the same proper length! Although Romain did not discriminate between the real and apparent positions of the rockets, it may be remarked that the trajectories A_2C_2, A_1C_1 in Fig.1 of Reference [22] actually represent the real positions of the rockets in S as discussed in Section 2 above. This figure is thus similar to Fig.1a of the present paper. Also the distance $B_2B_1^t$ represents, in S', the constant (real) separation of the rockets R1, R2 in their common co-moving inertial frame.

The apparent positions of the similarly accelerated rockets were later discussed by Evett [24]. No distinction was made between the real and apparent positions of the rockets and, unlike Romain, no definite acceleration procedure was defined. Like Dewan, a misinterpretation of the space time events corresponding to the firing of the rockets, as observed from their final inertial frame, lead to the erroneous conclusion that the real separation of the rockets in this frame is different from the initial one before acceleration.

What is essentially the 'Rockets-and-String Paradox' has been recently re-discussed by Tartaglia and Ruggiero [25], although no reference was made, in their paper, to the previous literature on the subject discussed in the present paper. As in Section 2 above, the case of two objects (spatially independent or connected by a spring), subjected to an idential uniform acceleration in their proper frames, as in Section 2 above, was considered, as well as that of an extended object (rod) subjected to a similar acceleration program. No distinction was made between the real and apparent positions of the objects considered, and no calculations using the space-time LT were performed. Exactly the same error as that of Dewan and Beran was made by introducing an incorrect distinction between the spatial interval between the ends of a connected rod and that between two spatially disconnected objects situated at the same spatial positions as the ends of the rod. Thus, it was concluded that the accelerated rod would be length contracted by an amount determined by its instantaneous velocity in S, as well as undergoing mechanical deformation due to elastic stress generated by the accelarating force. No calculations were performed, or arguments given, to justify this conclusion; it is simply stated that:

`The length seen by O is obtained from` l' `projectiong it from the x' axis (the space of O') to the x axis, i.e. multiplying by` $\sqrt{1-v^2/c^2}$`, according to the standard Lorentz contraction.`

Thus here, Tartaglia and Ruggiero were referring to the apparent, not the real, length of the rod. In the case of two independent, and similarly accelerated, objects, the real trajectories in S were correctly described, as in Eqns(2.14)-(2.22) above, and it was correctly stated that the separation of the objects remains constant in this frame. The real separation was, however, confused with the apparent separation between the objects, since it was then stated, again, without any supporting argument or calculation, that:

`.....the proper distance in the frame of the rockets,` l_0`, has progressively increased during the acceleration so that the Lorentz contraction (`$l = \gamma^{-1} l_0$`) produces precisely the` l `result.`

The general arguments and detailed calculations presented in Section 2 above clearly show that this statement is incorrect. There is no change in the separation of the objects in their co-moving inertial frame during the acceleration procedure. In the case when the two objects are connected by a spring, Tartaglia and Ruggiero incorrectly argued that the spring, being an extended object, undergoes Lorentz contraction whereas the spatial separation between the objects does not, thus inducing tension in the spring. It is then argued that:

`The consequence will be that the actual acceleration of the front end will be a little bit less than what the engine alone would produce and the acceleration at the rear end will be a little bit more for the same reason. In this way, the proper times of the two engineers will no longer be the same at a given coordinate time and the two world lines of the ends of the spring will no longer be equal hyperbolae (see figure 2).`

Since, in fact, there is no distinction for the LT, between the space interval between points on spatially separated objects, or an equal interval between points on a spatially extended object, there is no 'relativistic tension', generated in the spring, to give rise to the different accelerations of the two objects as conjectured in Reference [25]. Since the distances between the ends of the rod, the two spatially separated objects or the two connected objects remain always constant in the co-moving inertial frame, during the acceleration procedure, it follows that in all three cases considered in Reference [25], the real distance, in S, between the moving objects (except for some possible elastic deformation in the case of the accelerated rod) will be the same as that shown in Fig.1 of Reference [25]. This figure is similar to Fig. 1a of the present paper or Fig. 1 of Reference [22]. Fig. 2 of Reference [25] is therefore incorrect. Finally, it may be noted that the apparent length of a uniformly accelerated rod has been calculated by Nikolic [19]. The result obtained does not agree with the naive generalisation of the Lorentz contraction formula as given after Eqn(3) in Reference [25].

5 The Pole-and-Barn Paradox

For convenience, and without any loss of generality, the 'pole' of the example is replaced by two independent objects O1 and O2, separated by unit distance, that are symmetrically accelerated from rest in the inertial frame S (in which the barn is at rest), by the procedure described in Section 2 above, to the inertial frame S', co-moving with the objects. Units are chosen such that $c = 1$. Also, as in Fig.1, $L = a = 1$ and $t_{acc} = \sqrt{3}$. If the length of the barn is $0.5 + \delta$, where δ is a small number, then, due to the LFC, the apparent positions of O1 and O2, viewed in S, will, at some time, both fit into the barn. This example was introduced by Dewan [11], at the end of the paper written in reply to Nawrocki's objections to Dewan and Beran's 'Rockets-and-String paradox' paper. Dewan also imagined that the doors of the barn might be closed when the pole was inside, and that if the pole was suddenly stopped by the closed back door of the barn, the rod would

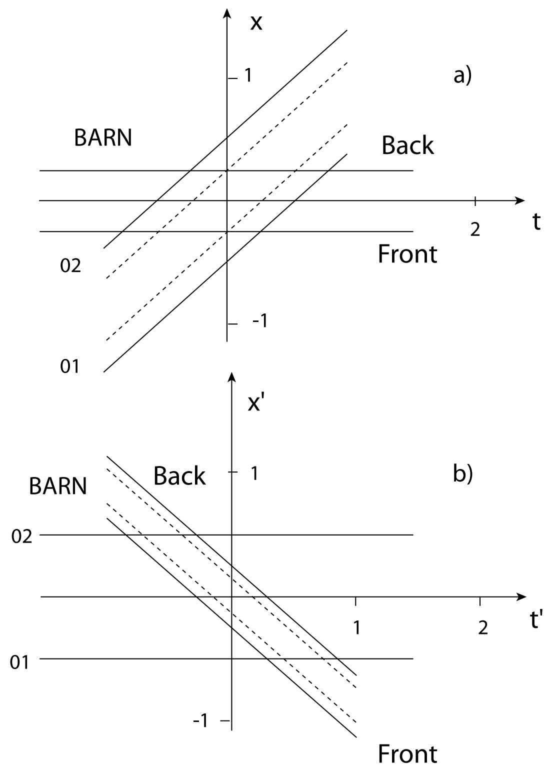

Figure 2: *Space-time trajectories of the real (full lines) and apparent (dashed lines) positions of O1 and O2 in the region of the barn. a) in S, b) in S'.*

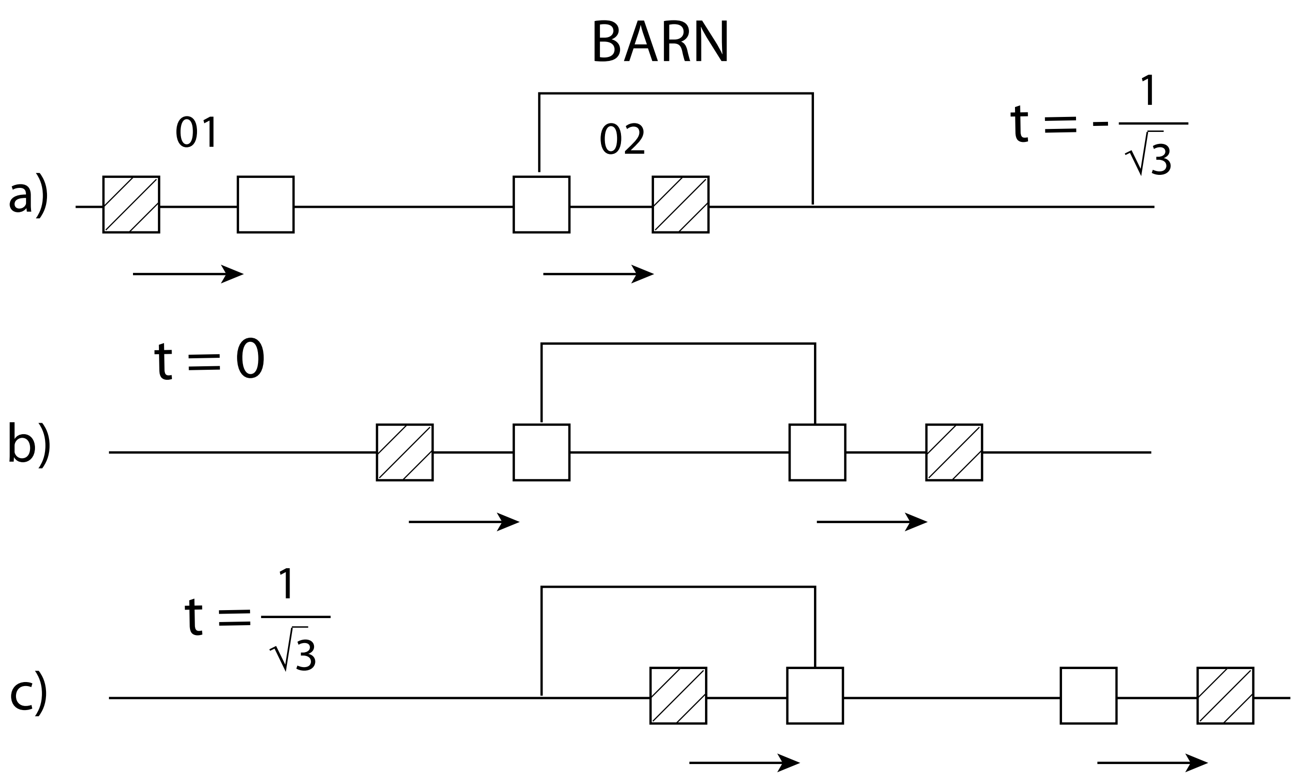

Figure 3: *Real (cross-hatched squares) and apparent (open squares) positions of O1 and O2 in S, as they pass by the barn.*

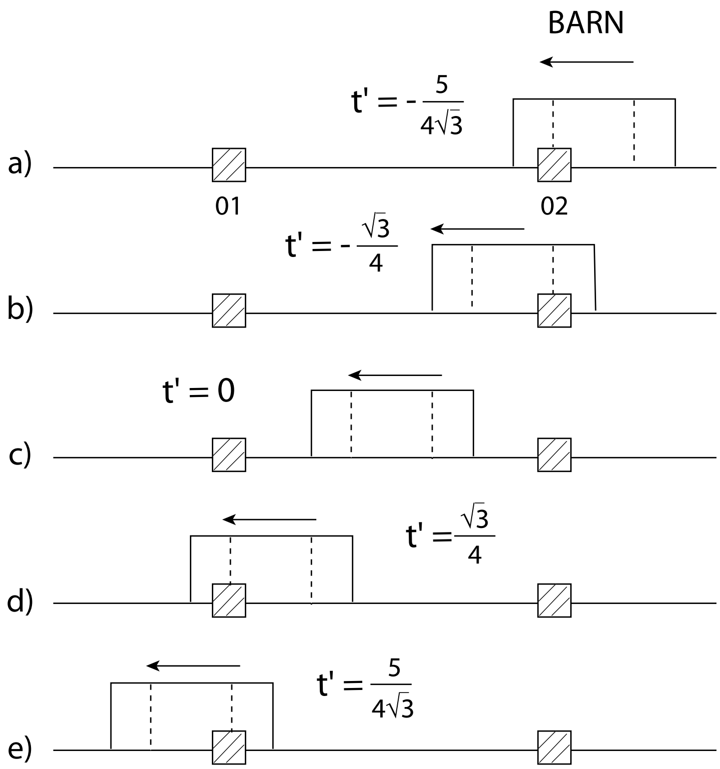

Figure 4: *The moving barn as observed from S'. The real and apparent positions of the ends of the barn are denoted by solid and dashed lines respectively.*

Time interval	O1	O2
$t \leq -\frac{1}{\sqrt{3}}$	OUT	OUT
$-\Delta \geq t > -\frac{1}{\sqrt{3}}$	OUT	IN
$\Delta > t > -\Delta$	IN	IN
$\frac{1}{\sqrt{3}} > t \geq \Delta$	IN	OUT
$t \geq \frac{1}{\sqrt{3}}$	OUT	OUT

Table 1: *Apparent positions (IN or OUT of the barn) of O1 and O2 as viewed in S. The barn is assumed to be slightly longer than the Lorentz-contracted distance between O1 and O2, so that $\Delta \ll 1$.*

Time interval	O1	O2
$t' \leq -\frac{5}{4\sqrt{3}}$	OUT	OUT
$-\frac{\sqrt{3}}{4} \geq t' > -\frac{5}{4\sqrt{3}}$	OUT	IN
$\frac{\sqrt{3}}{4} > t' > -\frac{\sqrt{3}}{4}$	OUT	OUT
$\frac{5}{4\sqrt{3}} > t' \geq \frac{\sqrt{3}}{4}$	IN	OUT
$t' \geq \frac{5}{4\sqrt{3}}$	OUT	OUT

Table 2: *Apparent positions (IN or OUT of the barn) of O1 and O2 as viewed in S'. The length of the barn is equal to the Lorentz contracted separation of the objects as viewed in S.*

Time interval	O1	O2
$t \leq -\frac{\sqrt{3}}{2}$	OUT	OUT
$-\frac{1}{2\sqrt{3}} \geq t > -\frac{\sqrt{3}}{2}$	OUT	IN
$\frac{1}{2\sqrt{3}} > t > -\frac{1}{2\sqrt{3}}$	OUT	OUT
$\frac{\sqrt{3}}{2} > t \geq \frac{1}{2\sqrt{3}}$	IN	OUT
$t \geq \frac{\sqrt{3}}{2}$	OUT	OUT

Table 3: *Real positions (IN or OUT of the barn) of O1 and O2 in S. The real positions in S' are given by the replacement $t \to t'$. The length of the barn is equal to the Lorentz contracted separation of the objects as viewed in S.*

revert to its original proper length, thus crashing through the closed front door of the barn (Figs.1 and 2 of Reference [11]).

In the associated 'paradox' Dewan supposes that a pole vaulter is running along carrying the pole:

```
The pole vaulter, on the other hand, would see the barn as contracted
and much smaller than the pole.  The question is, how can the pole
vaulter explain the fact that the door can be closed behind him?
```

All of the above takes for granted that the pole is really, not apparently, contracted in the frame S. However, the actual situation during the passage of the pole (i.e the objects O1 and O2) through the barn, is shown in Figs. 2, 3 and 4. In Figs.2a,b the real and apparent positions of O1 and O2, as a function of time, are shown in S and S' respectively. The clocks in S and S' are synchronised at the moment that the mid-point of O1 and O2 is situated at the middle of the barn. The real positions in S of O1 and O2, as calculated in Section 2 above, are indicated by the full lines, the apparent positions, due to the LFC, by the dashed ones. The real and apparent positions of the barn (at rest in S), as viewed from S' are similarly indicated. It is quite clear that at $t = t' = 0$, when the mid-point between O1 and O2 co-incides with that of the barn, O2 is already beyond the back door and O1 still before the front door. At no time are both objects inside the barn at the same time, so that the doors cannot be closed with them both inside. The passage of the objects through the barn is shown in more detail in Fig.3 (as viewed in S) and in Fig.4 (as viewed in S'). The real and apparent positions of the objects are shown as cross-hatched and open squares respectively. The times at which the objects apparently enter and leave the barn are shown in Table 1 for S and Table 2 for S'. Assuming that the barn is slightly longer than the Lorentz-contracted distance between the objects, it can be seen that, for a short period of time, both objects will appear to be inside the barn, as viewed from S, but that this is never the case for an observer in S'. Thus the apparent object 'barn with the doors closed and both objects inside' exists for an observer in S, but not for one in S'. This might be possible for the apparent positions, but must evidently lead to a contradiction if the 'barn with the doors closed and both objects inside' is associated with a real physical object, as was done by Dewan.

The paradox was discussed in Reference [14] where it was pointed out that, due to the relativity of simultaneity, and as can be seen in Fig.4 and Table 2, for an observer in S', O2 will leave the barn at $t' = -\sqrt{3}/4$ (Fig.4b) before O1 enters it at $t' = \sqrt{3}/4$ (Fig.4d). If the (apparent) doors of barn close at these instants (and the barn is slightly longer than 0.5 units) then O2 will be inside the barn when the front door closes, and O1 will be inside when back door closes, as also seen by an observer in S. It was not pointed out, however, in Reference [14] that the observer in S', unlike the one in S, never sees both objects in the barn at the same instant. The times at which the moving objects really enter and leave the barn in S are shown in Table 3. The corresponding times in S' are simply given by the replacement $t \to t'$.

In conclusion, the situation shown in Figs.1 and 2 of Reference [11] is a physically impossible one. The pole can never, at any time, really fit into the barn. As in the case of the Rockets-and-String paradox it has been incorrectly assumed that the apparent Lorentz contracted length of the pole in S is equal to its real length (as defined in Section 2 above) in this frame.

The real separation of the moving objects in S could be measured, as described in Section 2 above, by a parallel pair of light beams, perpendicular to the direction of motion of the objects, situated at the positions of the doors of the barn. Substituting the real times of passage of the objects given in Table 3 into Eqn.(2.13) gives their real separation: $L = 2\ell = 1$ unit.

6 The Lorentz-Fitzgerald Contraction and the Relativity of Simultaneity

It is interesting to consider the sequence of space-time events, in the frames S, S', which take place when the simultaneous observation is made in S of two objects, at rest in S', that yields the LFC. The correspondence, via the LT Eqns(3.1) and (3.2) of the relevant space-time events in S and S' is presented in Table 4 and shown in Fig.5.

When the mid-point of two objects O1 and O2 considered in the previous Section (the origin, O', of S') passes the center of the barn (corresponding to the origin, O, of S) the clocks in the frames are synchronised such that $t = t' = 0$. The apparent and real positions of the objects, at this time, in S are as shown in Fig.5b. E_1 and E_2 represent space-time events coincident with the observation of single photons from O1 and O2 respectively, that are both observed at $t = 0$ in S. According to the LT, the photon from O1 is emitted in S' at the later time $t' = \beta L/2c$ (event E'_1 in Fig.5c), and that from O2 at the earlier time $t' = -\beta L/2c$ (event E'_2 in Fig.5a), than the time ($t' = 0$) of observation of the photons in S. As before, L is the separation of the objects in S'. The position of the origin of S' in S measures directly t' via the relation, derived from Eqn(3.1), $t' = x^A_{O'}/\gamma v = x^R_{O'}/\gamma v$. As can be seen in Fig.5 and from the entries in Table 4, the real position of O' in S (mid-way between the real positions of O1 and O2) coincides with its apparent position, as calculated using Eqn(3.1), at all times. The real positions of O1 and O2 at the time, $t' = -\beta L/2c$, or $t = -\gamma\beta L/2c$, at which the observed photon (event E_2) is emitted from O2, are shown in Fig.5a. As above, $\gamma = 2$, $\beta = \sqrt{3}/2$, $L = c = 1$ are assumed. Similarly Fig.5c shows the real positions of the objects at the time $t' = \beta L/2c$, or $t = \gamma\beta L/2c$, at which the observed photon (event E_1) is emitted from O1. It can be seen from Fig.5b that

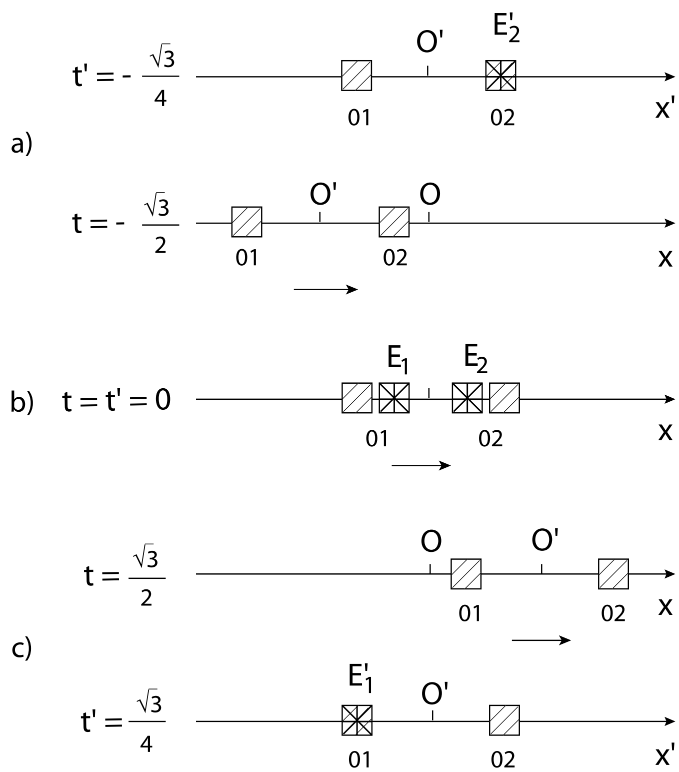

Figure 5: *Space-time events E_1' and E_2' in S' and the real positions of the objects O1 and O2 in S and S' corresponding to the simultaneous observation of E_1 and E_2 in S, that yields the LFC. a) at the time of photon emission from O2, b) at $t = t' = 0$, the time of observation in S of the LFC between E_1 and E_2 and c) at the time of photon emission from O1. The real positions of O1 and O2 are indicated by the cross hatched squares, and the emission (in S') or observation (in S) of photons by a star pattern.*

Object	x'	t'	x^A	x^R	t
01	$-\frac{L}{2}$	$\frac{\beta L}{2c}$	$-\frac{L}{2\gamma}$	$-\frac{L}{2}$	0
0'	0	$\frac{\beta L}{2c}$	$\frac{\gamma\beta^2 L}{2}$	$\frac{\gamma\beta^2 L}{2}$	$\frac{\gamma\beta L}{2c}$
02	$\frac{L}{2}$	$-\frac{\beta L}{2c}$	$\frac{L}{2\gamma}$	$\frac{L}{2}$	0
0'	0	$-\frac{\beta L}{2c}$	$-\frac{\gamma\beta^2 L}{2}$	$-\frac{\gamma\beta^2 L}{2}$	$-\frac{\gamma\beta L}{2c}$

Table 4: *Space-time points in S and S' of O1, O2 and O', at the times $t' = \beta L/2c$ and $t' = -\beta L/2c$ of photon emission in S' corresponding to observation of the LFC effect in S at $t = 0$. Real and apparent positions in S are denoted as x^R and x^A respectively. Note that there is distinction between real and apparent positions neither in S' nor, for the origin O', in S.*

the apparent position of O2 in S, at $t = 0$, is shifted, relative to its real position, towards negative values of x, i.e. in the direction of the real position of O2 at the instant that the corresponding photon was emitted in S'. Similarly, the apparent position of O1 in S, at $t = 0$, is shifted towards positive values, i.e. also towards its real position in S at the time that the corresponding photon was emitted in S'. The combination of these shifts gives the LFC. It is a direct consequence of the relativity of simultaneity of the events observed in S and S'. However, the apparent positions of the objects in S do not correspond to the real positions of the objects at the times that the photons were emitted in S', as might naively be expected. In fact it can be seen from the entries in Table 4 that at the time $t_2 = -\gamma\beta L/2c$, at which the photon observed at E_2 is emitted in S', the real position of O2 is:

$$x^R_{\rm O2}(t = t_2) = x^R_{\rm O'}(t = t_2) + \frac{L}{2} = \frac{L}{2}(1 - \gamma\beta^2) \quad (6.1)$$

which is separated from the apparent position of O2 at $t = 0$ by the distance:

$$x^A_{\rm O2}(t = 0) - x^R_{\rm O2}(t = t_2) = \frac{L}{2}(\frac{1}{\gamma} + \gamma\beta^2 - 1) = \frac{L}{2}(\gamma - 1) \quad (6.2)$$

The same distance separates the apparent position of O1 at $t = 0$ from its real position at the time, $t_1 = \gamma\beta L/2c$, at which the photon was emitted from O1 in S'. Table 4 and Fig.5 show that the photon from O2 is predicted by the LT to be observed in S at a time $\gamma\beta L/2c$ later than that corresponding to its emission time in S', whereas the photon from O1 is predicted to be observed in S at a time $\gamma\beta L/2c$ *before* O1 reaches the position at which the photon is emitted. The latter conclusion seems indeed paradoxical [26] and merits further reflection.

A similar consideration of the Space Dilatation effect shows that this is also follows directly from the relativity of simultaneity. Making a $\Delta t' = 0$ projection in S' gives corresponding non-simultaneous events in S, spatially separated by the distance γL (see

Table 1 of Reference [7]).

7 Summary and Conclusions

The real positions of moving objects within a single frame of reference are calculated by the well-known rules, based on the concepts of spatial position, time, velocity and acceleration, as first clearly set down by Galileo. Special Relativity which instead relates, via the LT, space-time events in different inertial frames, has no relevance to the calculation of the relative positions of different moving objects in a single reference frame. The different apparent sizes of extended objects, or distances between discrete objects, predicted by the LT for different modes of observation, e.g. the LFC ($\Delta t = 0$ projection) or SD ($\Delta t' = 0$ projection) manifest the truly apparent (i.e. observation mode dependent) nature of these phenomena.

The incorrect conclusion concerning the Rockets-and-String paradox presented till now in the literature resulted from, on the one hand, a failure to distinguish between the (correctly calculated) real positions of the rockets and the apparent nature of the LFC, and on the other, by application of the LFC in an inconsistent manner, either to the string and not to the rocket separation in S, or to the rocket separation, but not to the string, in S'. There are no 'stress effects' specific to SR and so the string does not break.

Similarly, all discussions of the Pole-and-Barn paradox (and several similar examples cited above), in the literature and text books, have assumed that the LFC is a real, not an apparent, effect. There seems to be no justification for such an assumption in Einstein's original work on SR. When the real positions of the moving objects (or rod) in S are properly taken into account it is evident that they can never actually fit into the barn so that the situation depicted in Figs.1 and 2 of Reference [11] represents a physical absurdity.

It is shown that both the LFC and SD effects are direct consequences of the relativity of simultaneity of SR first proposed by Einstein. In the case of the LFC, what seems to be a true paradox is revealed by this study. A photon, emitted by a moving object, is predicted, by the LT, to be observed before the moving object has reached the position at which it emits the photon. Essentially the same apparent breakdown of causality as evidenced by 'backwards running clocks' has been pointed out in a recently published paper [26].

Acknowledgements

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