Proposals for Two Satellite-Borne Experiments to Test Relativity of Simultaneity in Special Relativity

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Abstract

An orbiting 'photon clock' is proposed to test directly the relativity of simultaneity effect of special relativity. This is done by exchanging microwave signals between two satellites in low Earth orbit carrying clocks that have previously been synchronised in the Earth-centered inertial system. A similar experiment using synchronised signals from two GPS satellites with the receiver on a single satellite in low Earth orbit, is also proposed.

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1 Introduction

In Einstein's original 1905 Special Relativity (SR) paper [1], three novel space-time phenomena were introduced as physical consquences of the Lorentz transformation, i.e. as purely kinematical effects. These are: Relativity of Simultaneity (RS)[1] relativistic Length Contraction (LC) and Time Dilation (TD). Many direct and precise experiments have confirmed the existence of TD; however to date, there is no experimental confirmation of either RS or LC. It is the purpose of the present article to propose definitive experimental tests of RS.

Unlike essentially all recent high precision texts of special relativity, the aim of the proposed experiments is not to search for a breakdown of SR but rather to test a particular prediction of standard SR. This important point was not understood by some reviewers of earlier versions of this paper who stated, incorrectly, that existing experimental limits on SR test parameters (for example those of the Mansouri and Sexl test theory [2]) show that SR is experimentally confirmed, so therefore there is 'nothing to test'. For this reason, in the following section, a 'mini-review' of current experimental tests is presented, where the complete lack of overlap between conclusions drawn from the results of previous experiments, and those that may be obtained from the proposed experiments, is pointed out. The standard relativity of simultaneity effect of SR is explained in Section 3. The following sections present the proposed experiments and a comparative discussion of the RS test proposed by Fürth [3] in 1965. A brief summary is provided in the concluding section.

2 Mini-review of experimental tests of special relativity

The availability of precise atomic clocks and frequency stabilised lasers has enabled an enormous increase, over the last four decades, in the precision of of some experimental tests of SR. In particular, much improved experimental upper limits on the anisotropy of the speed of light, testing Local Lorentz Invariance (LLI), have accompanied the development of sophisticated theoretical models such as the 'standard model extension' [4] as well as more specific models employing field- [5] or string- [6] theoretical ideas. The most widely used test theory for SR and LLI is, however, the kinematical one of Robertson [7] and Mansouri and Sexl [2](RMS). This model assumes the existence of a 'preferred frame', Σ, (often identified with the proper frame of the cosmic microwave background) relative to which an arbitary inertial frame, S, is in motion with velocity $\vec{v} = c\vec{\beta}$. Denoting time and spatial position in Σ as $(T;\vec{X})$ and in S as $(t;\vec{x})$, and assuming conventional Einstein light-signal clock synchronisation, the model postulates the following space time transformation equations:

$$x = b(X - vT), \tag{2.1}$$

[1]In fact, the RS effect discussed in Ref. [1] is not the one described below in Section 3, that is correlated with the LC effect, but follows from a discussion of light signal clock synchronisation in different inertial frames, without reference to the Lorentz transformation. The RS effect tested by the proposed experiments is the standard text-book one derived, like LC, from the Lorentz transformation.

$$t = (a + b\beta^2)T - \frac{b\beta X}{c}, \quad (2.2)$$

$$y = Yd, \quad z = Zd \quad (2.3)$$

where, in SR,

$$b = \frac{1}{a} = \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad d = 1. \quad (2.4)$$

in which case (2.1)-(2.3) become the Lorentz transformation (LT) and Σ another inertial frame. Since in most tests of SR and LLI the leading relativistic terms are of $O(\beta^2)$, it is convenient to define further parameters $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\delta}$, such that:

$$a = 1 + \tilde{\alpha}\beta^2 + ..., \quad b = 1 + \tilde{\beta}\beta^2 + ..., \quad d = 1 + \tilde{\delta}\beta^2 + ... \quad (2.5)$$

In SR $-\tilde{\alpha} = \tilde{\beta} = 1/2$, $\tilde{\delta} = 0$.

The most precise tests of the isotropy of the speed of light are modern versions of the Michelson-Morley [8](MM) and Kennedy-Thorndike [9](KT) experiments. The MM experiment tests the angular dependence of light speed anisotropy. A recent limit [10], expressed in terms the parameters of the RMS test model is:

$$\tilde{\beta} - \tilde{\delta} - \frac{1}{2} = (-2.1 \pm 1.9) \times 10^{-10}$$

The KT experiments are sensitive to the velocity dependence of light speed anisotropy, recent results [11] giving:

$$\tilde{\beta} - \tilde{\alpha} - 1 = (-3.1 \pm 6.9) \times 10^{-7}$$

The parameter $\tilde{\alpha}$ is measured in experiments (successors of that of Ives and Stillwell [12]) on time dilation or the transverse Doppler shift. An experiment using collinear saturation spectroscopy of optical transitions in a beam of $^7Li^+$ with $\beta = 0.064$ [13] obtains the limit:

$$\tilde{\alpha} + \frac{1}{2} < 2.2 \times 10^{-7}$$

Another experiment sensitive to $\tilde{\alpha}$, of particular interest, in view of the second experiment proposed in the present letter, uses the GPS satellite system to test for anisotropy of the one-way speed of light [14]. It was found that $\delta c/c < 2 \times 10^{-9}$ corresponding to the limit:

$$|\tilde{\alpha} + \frac{1}{2}| < 10^{-6}$$

The RMS test model parameter a is called the 'time dilation' parameter. That this nomenclature is correct is evident from Eqns(2.1) and (2.2). Suppose a clock is at rest at $x = 0$ in S. Eqn(2.1) shows that, in Σ, it has the space-time trajectory $X = vT$. Substituting this relation in (2.2) gives immediately $t = aT$, which becomes, in SR, $T = \gamma t$ which is indeed the TD effect. The parameter b is commonly called the 'length contraction parameter' which looks plausible in view of Eqn(2.1). The LC effect is however more subtle than simple inspection of Eqn(2.1) might suggest. Two space-time events and two clocks are involved, and the synchronisation of distant clocks is crucial. This means that the limit on $\tilde{\beta}$ provided by the MM or KT experiments does not confirm or invalidate the existence of LC. The relation between the RS and LC effects [15] and the question of their existence [16] have been previously discussed by the present author. Reference [16] also contains a concise review of the experimental tests, to date, of SR. In any case, if it does exist, LC is certainly an O(β^2) effect, and the experiments proposed here, to test for RS, are quite insensitive to it.

3 The relativity of simultaneity effect

To discuss RS it is convenient to use what Mansouri and Sexl call 'system external synchronisation' [2]. Suppose that there are clocks at $x = 0$ and $x = l$ in the frame S. Two clocks at rest in Σ, spatially contiguous with the clocks in S at some instant, are simultaneously synchronised with those in S. That is both clocks in Σ are set to $T = 0$ and both clocks in S to $t = 0$. The transformation describing the relation between the positions and times in Σ and S of the clock at $x = 0$ is then given, in the RMS test model, by Eqns(2.1) and (2.2) with $x = 0$, whereas the corresponding equations for the clock at $x = l$ are instead:

$$x - l = b(X - L - vT), \tag{3.1}$$

$$t = (a + b\beta^2)T - \frac{b\beta(X - L)}{c}. \tag{3.2}$$

Substituting $x = l$, $t = T = 0$ in (3.1) and (3.2) shows that these equations are verified providing that $X = L$ at this instant. At the later time T, in the frame Σ, Eqns(2.1) and (2.2) give, for the time of the clock in S at $x = 0$, $t(x = 0) = aT$, while (3.1) and (3.2) give for the time of the clock at $x = l$ in S, $t(x = l) = aT$. Therefore, in SR:

$$t(x = 0) = t(x = l) = \frac{T}{\gamma}. \tag{3.3}$$

Both clocks show the same TD effect, but there is no RS[2]. If now the substitution $x = l$ is made in (2.2), instead of (3.2), the relation between T and $t(x = l)$ and T is found to be:

$$t(x = l) = (a + b\beta^2)T - \frac{b\beta X(x = l, T)L}{c}. \tag{3.4}$$

So that instead of (3.3) it is found in SR that

$$t(x = 0) = t(x = l) + \frac{\gamma\beta L}{c} = \frac{T}{\gamma} \tag{3.5}$$

where $L \equiv X(x = l, T) - X(x = 0, T)$ This means that, at any epoch, T, in the frame Σ, the clocks in S at $x = 0$ and $x = l$ show times differing by $\gamma\beta L/c$. That is, events that are simultaneous in Σ are not so, according to the clocks in S. Eqn(3.5) corresponds to the standard text-book presentation of RS [17]. Note that both (3.3) and (3.5) are predictions of SR. They differ only in the manner in which the spatial coordinates of the clock at $x = l$ are inserted into the Lorentz transformation equations. The possible

[2]The same physical effect has been considered by Mansouri and Sexl as the consequence of a specfic synchronisation procedure of the clocks in the frame S. See Eqns(3.6) of the first reference in [2].

correctness of (3.3) does not therefore mean that $a \neq 1/\gamma$ or $b \neq \gamma$ in (3.4). In both cases $a = 1/b = 1/\gamma$. Eqns(3.3) and (3.5) differ, not in the parameters of the test model, but by a different description of initial conditions in the LT equations, without any violation of LLI. The two experiments proposed below can clearly distinguish between the two possibilities (3.3) (no RS) and (3.5) (RS exists). It is important for this that the RS time difference in (3.5), $\gamma\beta L/c$, is an O(β) not an O(β^2) effect.

4 An Einstein light-signal clock in space

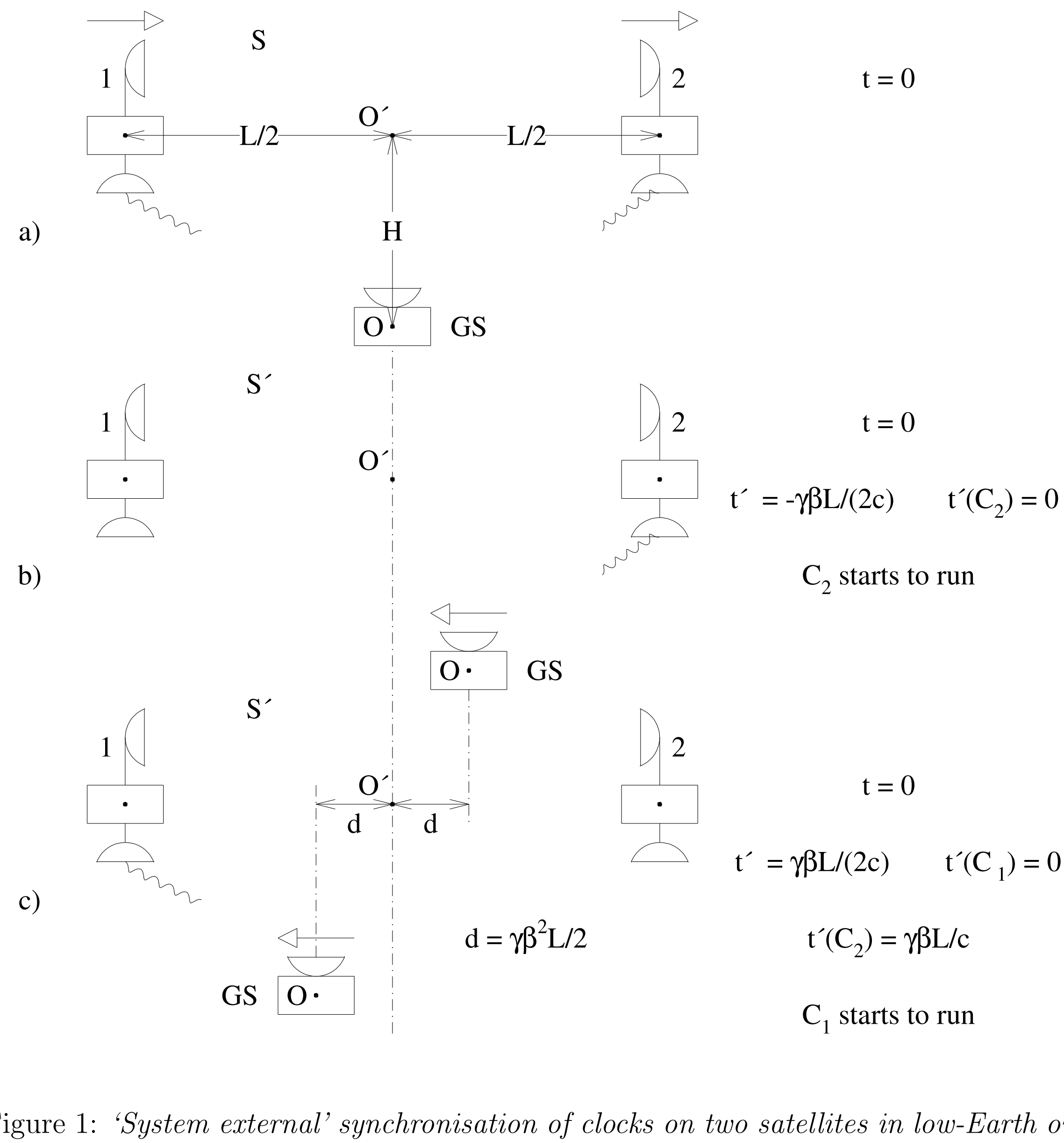

Figure 1: *'System external' synchronisation of clocks on two satellites in low-Earth orbit. a) Simultaneous microwave signals are sent from the ground station GS so as to arrive simultaneously in the ECI frame at Satellites 1 and 2. b) Signal arrives at Satellite 2 in the comoving inertial frame S' of the satellites and starts clock C_2. c) Signal arrives at Satellite 1 in S' and starts clock C_1 The configurations in b) and c) are calculated using Eqs. (2.1) and (2.2) with $b = 1/a = \gamma$, $d = 1$. See text for further discussion.*

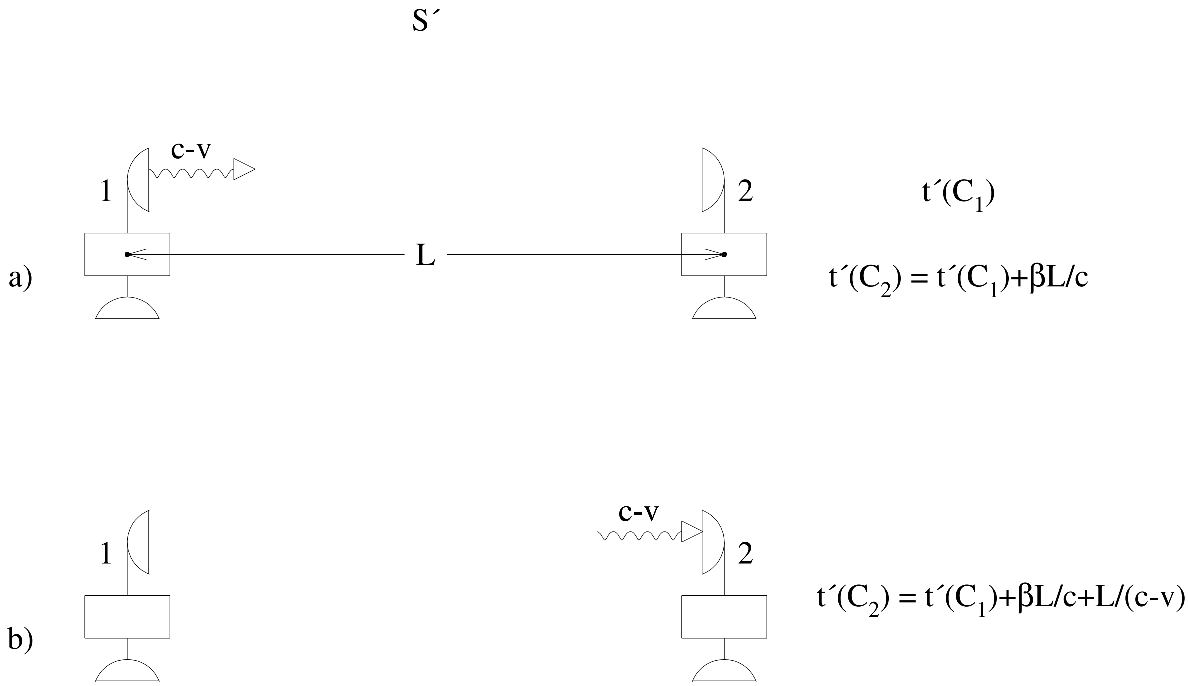

Figure 2: *Exchange of a microwave signal between the previously synchronised Satellites 1 and 2 as viewed in the common comoving inertial frame S' of the satellites. a) The signal is sent at epoch* $t'(\mathrm{C}_1)$ *from Satellite 1 and received [b)] at epoch* $t'(\mathrm{C}_2)$ *by Satellite 2.* $O(\beta^2)$ *corrections are neglected. See text for further discussion.*

The first experiment proposed makes use of the light-signal clock synchronisation procedure described by Einstein in [1]. The proposed experiment requires two satellites, one of which could conveniently be the International Space Station (ISS) the other a satellite, following the same orbit as the ISS, but separated from it by a few hundred kilometers.

A scheme of the proposed experiment is shown in Fig.1. The satellites, 1 and 2, in low Earth orbit, separated by the distance L, pass near to a ground station GS. Cartesian coordinate systems are defined in the co-moving inertial frame of the satellites (S') and the ground station (S). The origin of S' is chosen midway between 1 and 2 with x'-axis parallel to the direction of motion of the satellites and y' axis outwardly directed in the plane of the orbit. Ox and Oy are parallel to Ox' and Oy' at the position of closest approach (culmination) of O' to GS At culmination, the coordinates of O' in S are: (x,y,z) = (0,H,D) and the relative velocity of frames S and S' is parallel to the x-axis of magnitude $v = c\beta$[3]. As shown in Fig. 1a, clocks C_1, C_2 in the satellites 1, 2 are synchronised in the frame S at time $t = 0$ by reception of simultaneous microwave signals sent from GS at such a time as to arrive simultaneously at the satellites when O and O' are aligned. The corresponding times of arrival in the frame S' as predicted by the usual Lorentz transformation equations ((2.1) and (2.2) with $x = x'$, $t = t'$, $X = x$, $T = t$, $b = 1/a = \gamma$ and $d = 1$) are $t' = -\gamma\beta L/(2c)$ for Satellite 1 and $t' = \gamma\beta L/(2c)$ for Satellite 2. On reception of the signals, the identical clocks C_1 and C_2, which are previously stopped and set to record an epoch zero, are started. After reception of the signal by Satellite 1 the epochs recorded by the clocks at any instant in the frame S' therefore satisfy the relation:

$$t'(\mathrm{C}_2) = t'(\mathrm{C}_1) + \frac{\gamma\beta L}{c}. \tag{4.1}$$

The configurations of the clocks and the ground station, as viewed in the frame S', on reception of the synchronistion signals by Satellites 2 and 1 respectively, are shown in Fig. 1b and 1c, together with the epochs registered by the running clocks.

In order to test the RS effect that is manifest in Eq. (4.1) it is sufficient to send, as in Einstein's clock synchronisation procedure, a light signal from from Satellite 1 to Satellite 2 (or vice versa) at a known epoch of the clock on the signal-emitting satellite and observe its time of reception at the other one. As in the analysis of the operation of the GPS system [21, 23] it is important to take properly include the Sagnac effect [24, 25] in the calculation of the flight time of the signal. This is done by taking into account the relative velocity of the light signal, which is assumed to move at the universal speed c in the Earth-Centered (ECI) frame, and the signal receiver. The ECI frame is instantaeously comoving with the centroid of the Earth, with origin at the latter, and has coordinate axes with constant orientations relative to the fixed stars. It is the frame in which the synchronisation of the clocks in the satellites of the GPS system is defined [21]. Since the speed of the ground station in the ECI frame due to the rotation of the Earth ($<$ 0.5km/s) is considerably less than that of a low-Earth-orbit satellite (7.7km/s for the International Space Station (ISS) [18]) the relative velocity, v, of the frames S and S' is, to a good

[3]It is assumed in the calculations that the change of the co-moving inertial frame due to the actual, nearly circular, orbital motion of the satellites may be neglected for small values of t and t'. With an inter-satellite distance of 400 km the satellites move about 11 m during the transit time of the signal between them. This corresponds to a change in the direction of the comoving system of 1.6 μrad and a lateral shift of the reciever position of only 17 μm.

approximation, equal to that of the satellite relative to the ECI frame that is needed to calculate the Sagnac effect.

The exchange of a light signal between Satellite 1 and Satellite 2 as viewed in their common comoving frame S' is shown in Fig. 2. If the signal is emitted at time $t'(\mathrm{C}_1) = \tilde{t}'_1$ the predicted time of arrival as recorded by the clock C_2 is given by (4.1) and the Sagnac effect as $t'(\mathrm{C}_2) = \tilde{t}'_2$ where

$$\Delta t'_{\mathrm{RS}} \equiv \tilde{t}'_2 - \tilde{t}'_1 = \frac{L'}{c-v} + \frac{\beta L'}{c} = \frac{L}{c} + \frac{2\beta L}{c} + \mathrm{O}(\beta^2) \qquad \text{(RS)} \tag{4.2}$$

where $L' \equiv \gamma L$. In the absence of the RS effect (i.e. employing Eqs. (3.1) and (3.2) to give the SR prediction instead of Eqs. (2.1) and (2.2)) it is found that

$$\Delta t'_{\mathrm{No\ RS}} \equiv \tilde{t}'_2 - \tilde{t}'_1 = \frac{L}{c-v} = \frac{L}{c} + \frac{\beta L}{c} + \mathrm{O}(\beta^2). \qquad \text{(No RS)} \tag{4.3}$$

Comparing (4.2) and (4.3) it is seen that

$$\Delta t'_{\mathrm{RS}} - \Delta t'_{\mathrm{No\ RS}} = \frac{\beta L}{c}. \tag{4.4}$$

If the satellites have the orbital velocity of the ISS, $v = 7.67$ km/s ($\beta = 2.56 \times 10^{-5}$) and choosing $L = 400$ km, the time-of-flight of the microwave signal between the satellites is $\simeq 1.4$ ms and $\beta L/c = 34.2$ ns to be compared with a time resolution of better than 1 ns in, for example, the NAVEX low Earth orbit satellite experiment [19]. The actual distance $\mathcal{L}$ separating the satellites during the measurement of $\Delta t'$ can conveniently be found by measuring the time of arrival at Satellite 1, $t'(\mathrm{C}_1) = \tilde{t}'_{1R}$, of the signal reflected back from Satellite 2. Then

$$\tilde{t}'_{1R} - \tilde{t}'_1 = \frac{\mathcal{L}}{c-v} + \frac{\mathcal{L}}{c+v} = \frac{\mathcal{L}}{c(1-\beta^2)} \tag{4.5}$$

so that

$$\mathcal{L} = \frac{c(1-\beta^2)(\tilde{t}'_{1R} - \tilde{t}'_1)}{2} = \frac{c(\tilde{t}'_{1R} - \tilde{t}'_1)}{2} + \mathrm{O}(\beta^2) \tag{4.6}$$

and

$$\Delta t'_{\mathrm{RS}} = \frac{(1+2\beta)}{2}(\tilde{t}'_{1R} - \tilde{t}'_1) + \mathrm{O}(\beta^2), \tag{4.7}$$

$$\Delta t'_{\mathrm{No\ RS}} = \frac{(1+\beta)}{2}(\tilde{t}'_{1R} - \tilde{t}'_1) + \mathrm{O}(\beta^2). \tag{4.8}$$

The ease of measurement of the $\mathrm{O}(\beta)$ RS effect, evident from the predicted results of the experiment just described, may be contrasted with the difficulty of measuring, in a similar experiment, the $\mathrm{O}(\beta^2)$ LC effect. Using the value $\beta = 2.56 \times 10^{-5}$ appropriate for the ISS, the apparent contraction of the distance between the satellites 1 and 2, as viewed at some instant in S, of $(1 - 1/\gamma)L$, amounts to only 131μm for $L = 400$ km. It is hard to see how any experiment using currently known techniques could have a sufficiently good spatial resolution to measure such a tiny effect.

5 Comparison with the experiment proposed by Fürth to detect RS

In this section the experiment described above is compared with that proposed by Fürth in 1965 [3]. In the latter, the phase difference between signals from two spatially-separated phase-synchronised microwave transmitters as measured by a receiver, either at rest or in motion relative to the transmitters, was considered. Since the phase of an electromagnetic wave is unchanged by propagation in free space, the phase difference $\Delta\phi_0$ measured by the stationary receiver is given by the time-of-flight of the signal from the further transmitter to the nearer one: $\Delta\phi_0 = 2\pi\nu l/c$. Here l is the separation of the transmitters parallel to the direction of observation and ν is the microwave frequency. If the receiver is in uniform motion, parallel to the direction of the microwave signals, with speed v relative to the transmitters, use of the conventional LT predicts, as in Eq. (3.5), an additional phase difference due to the RS effect:

$$\Delta\phi_v = \Delta\phi_0 + \frac{2\pi\nu l v}{c} \quad (c' = c - v) \tag{5.1}$$

where only the lowest order, $\mathrm{O}(\beta)$, velocity-dependent term is retained, and the relative speed c' of the signals and the receiver, in the rest frame of the latter, due to the Sagnac effect, is $c' = c - v$.

It was argued by Berry *et al* [20] that the phase shift $\Delta\phi_0$ in the above equation, corresponding to the passage of the signal between the two transmitters would be modified, at $\mathrm{O}(\beta)$, in the rest frame of the moving receiver. Assuming that the light signals move at the same speed c in the rest frames of both the transmitters and the receiver, as predicted by the conventional Relativistic Parallel Velocity Addition Relation (RPVAR): $w = (u+v)/(1+uv/c^2)$ of SR, the relative velocity between the signals and the transmitters in the rest frame of the receiver is found to be $c+v$ This gives a different prediction for $\Delta\phi_v$:

$$\Delta\phi_v = \frac{2\pi\nu l}{c+v} + \frac{2\pi\nu l v}{c} = \Delta\phi_0 + \mathrm{O}(\beta^2). \quad (c' = c) \tag{5.2}$$

At $\mathrm{O}(\beta)$ the phase shift due to RS is cancelled by a term arising from the different signal flight time in the receiver rest frame. This calculation [20] does not however take correctly into account the lowest order Sagnac effect. At $\mathrm{O}(\beta)$ the relative velocity of the signals and the transmitters is *the same* in the rest frame of the transmitters and that of the receiver. If this were not the case the experimentally confirmed Sagnac effect simply would not occur. If then the experiment proposed by Fürth were to be performed, and the same phase shift measured for stationary and moving receivers, this is evidence that the RS effect does not exist, not as claimed in Ref. [20] that this is the correct prediction of standard SR with RS and the conventional RPVAR. If the latter were applicable in the experiment under consideration then the Sagnac effect would not occur.

There is a close similarity between the experiment proposed in the previous section of the present paper and that proposed by Fürth. In the former the clocks are synchronised in a frame in which they are in uniform motion and then later compared in their common comoving frame. In the latter, microwave transmitters (effectively clocks) are synchronised in their common rest frame and then compared in a frame in which they are in motion.

In both cases time (or phase) differences are generated by signal propagation between the clocks (or transmitters). In both experiments the relative speed (at lowest order in v) of the signals and their receiver is $c-v$. Setting (incorrectly) the relative speed of the signals and the receiver to c, in the rest frame of the receiver, in either experiment, yields similar predictions. Instead of (4.2) it is found that

$$\Delta t'_{\mathrm{RS}} = \frac{L}{c} + \frac{\beta L}{c} + \mathrm{O}(\beta^2) \quad (c' = c) \tag{5.3}$$

and instead of (4.3)

$$\Delta t'_{\mathrm{No\ RS}} = \frac{L}{c} + \mathrm{O}(\beta^2) \quad (c' = c) \tag{5.4}$$

However (4.4) remains unchanged:

$$\Delta t'_{\mathrm{RS}} - \Delta t'_{\mathrm{No\ RS}} = \frac{\beta L}{c} + \mathrm{O}(\beta^2) \quad (c' = c) \tag{5.5}$$

The prediction $\tilde{t}'_2 - \tilde{t}'_1 = L(1+\beta)/c$ is that of the Sagnac effect in the absence of RS (Eq. (4.3)), or alternatively (Eq.(5.3)) that of conventional SR (RS and the RPVAR). In the latter case the experimentally confirmed Sagnac effect is also predicted to not exist, so that the prediction, $c' = c$, of the conventional RPVAR is untenable. Experimental verification of the relation (4.3) then unambigously demonstrates the non-existence of the RS effect.

6 Relativity of simultaneity test using GPS satellites: Confusion and consternation

An interesting variant of the experiment described in Section 4 above uses the synchronised satellite-borne clocks of the Global Positioning System (GPS). The latter provide, after correction of their proper times for the effects of relativistic time dilation and gravitational blue shift, a coordinate time, t_C, defined in the ECI reference frame [21]. Associating, as before, the ECI frame with S (coordinates (t; x, y, z) and the comoving inertial frame of a single satellite, R, in low Earth orbit, with the frame S' with coordinates (t'; x', y', z') the scheme of the proposed experiment is shown in Fig. 3. It is assumed, for simplicity, that the satellite's orbit is in the same plane as the orbits of a constellation of four GPS satellites GPS1-GPS4. In the experiment, signals are sent simultaneously, in the ECI frame, from GPS1 and GPS2, at such a time as to arrive at R when it is in the position, visible from the satellites, at which $x = 0$. This corresponds, in GPS nomenclature, to 'transmitter time tagging' [21]. The configuration in the frame S at the instant that the signals are sent is shown in more detail in Fig. 4a, while configurations in the rest frame, S', of R calculated according to Eqs. (2.1) and (2.2) for the SR case, at the times when the signals are emitted, in this frame, from GPS2 and GPS1 are shown in Fig. 4b and 4c respectively. If s is the flight distance of the microwave signals in the frame S, the satellite R is at $x = -\beta s$ in the frame S at the time of their emission. At the emission time of the signals the origins of S and S' are aligned in x, and $t = 0$. The signal from GPS2 is emitted at time $t'_2 = -\gamma\beta L/(2c)$ at an angle θ'_2 to the x'-axis, where $\tan\theta'_2 = H/[\gamma(L/2+\beta s)]$ (Fig. 4b). The signal from GPS1 is emitted at the later time,

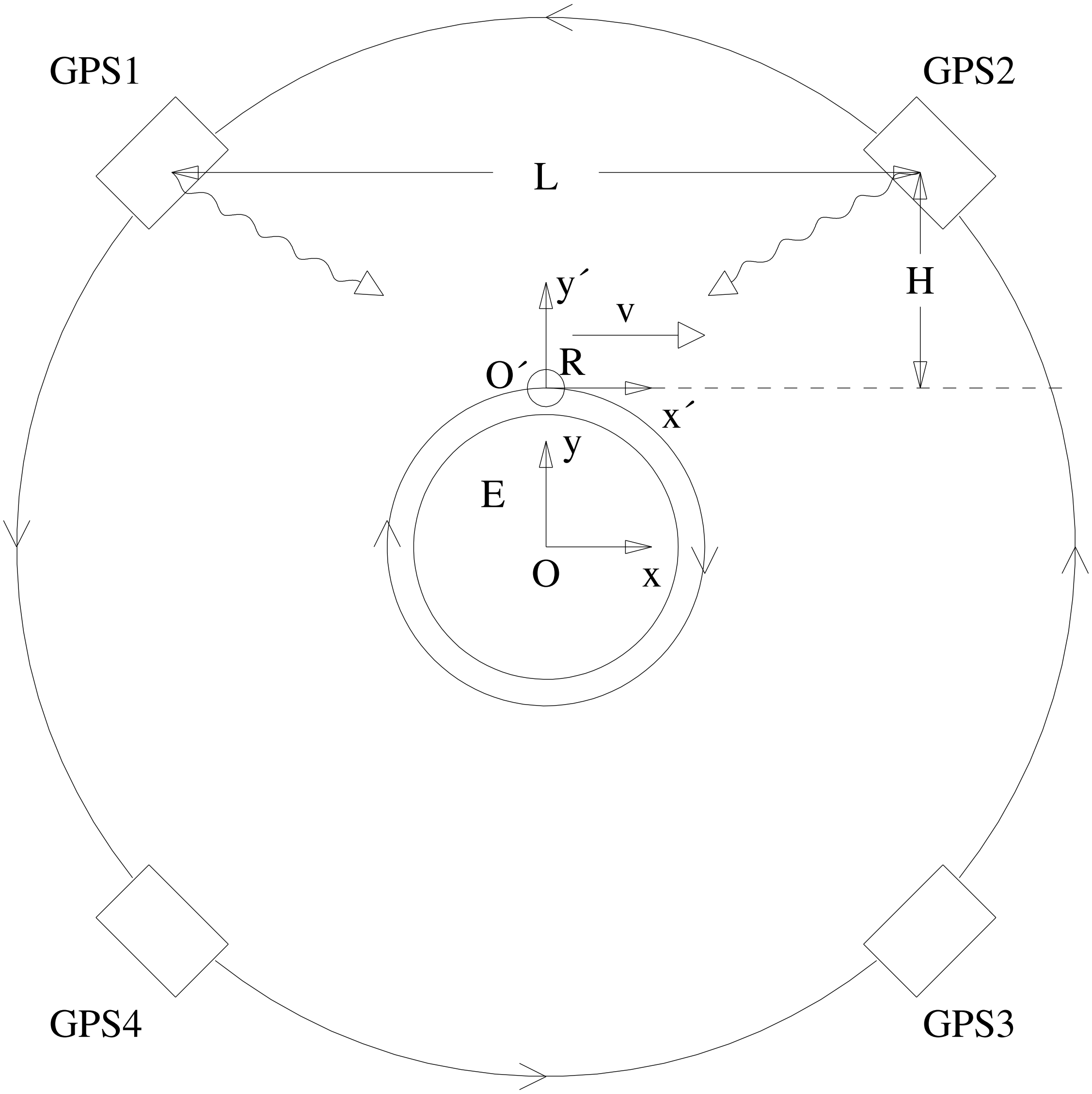

Figure 3: *Test of relativity of simultaneity using GPS satellites as transmitters and a satellite, R, in low Earth orbit as receiver. The sizes of the Earth, E, and the orbit of the GPS satellites are shown approximately to scale; for clarity the size of the orbit of R is enlarged. Signals emitted synchronously in the ECI frame (S') by GPS1 and GPS2 are received at R when it is at* $x = 0$. *According to the relativity of simultaneity of standard special relativity, the signals are received, in the proper frame, S', of R, with a time difference of* $\Delta t = vL/c^2$, *where* v *is the velocity of R relative to the ECI frame. The figure shows the spatial configuration as seen by an observer in the latter frame at the instant that the signals are sent.*

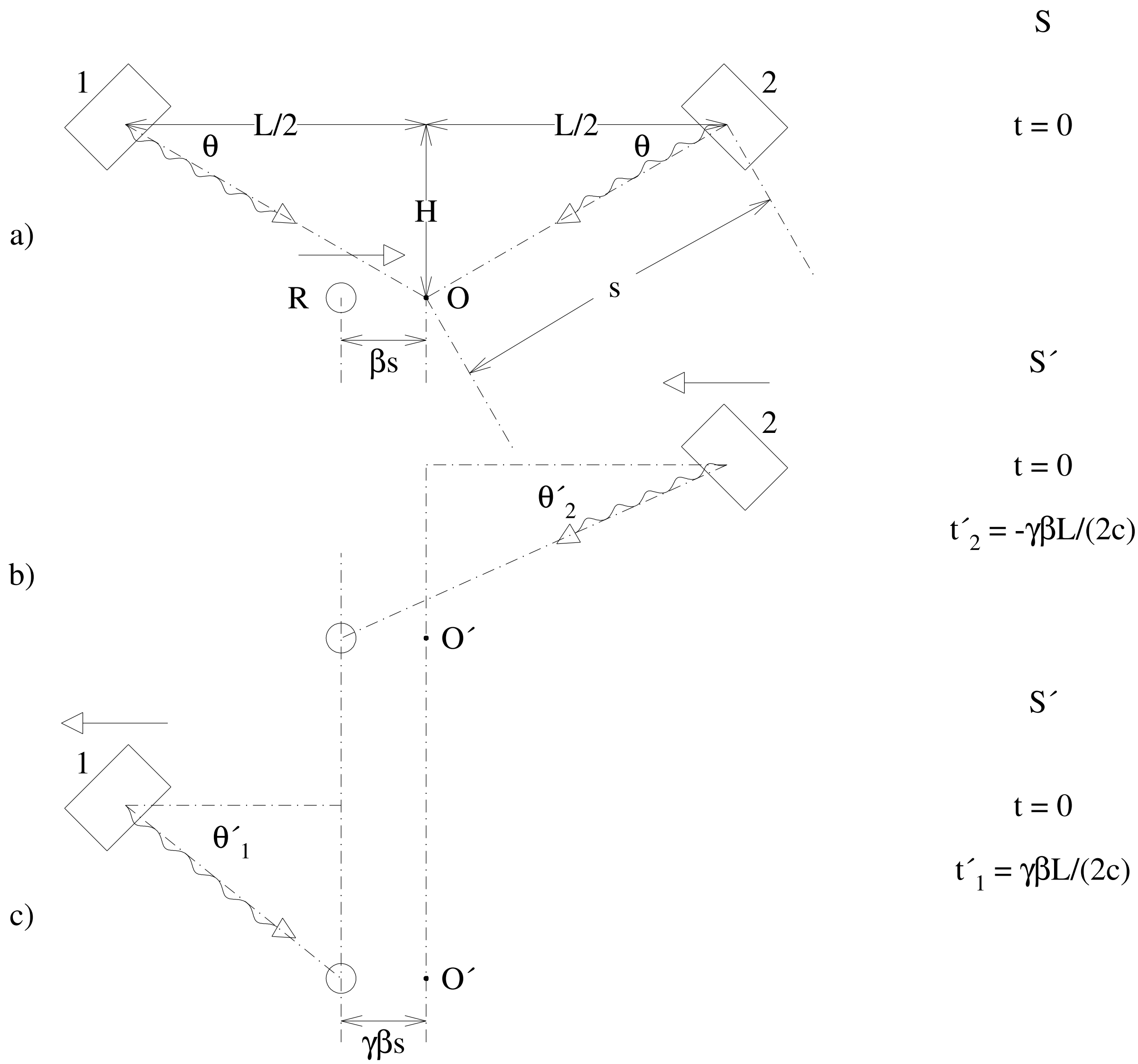

Figure 4: *Spatial configurations in different frames at the instant of emission of the microwave signals from the satellite GPS1 (1) and GPS2 (2) in the experiment shown in Fig. 3. a) in the ECI frame S, b) in the comoving inertial frame S', of the satellite R, at the time of emission of the signal from 1. and c) in S' at the time of emission of the signal from 2.The configurations in b) and c) are calculated using Eqs. (2.1) and (2.2) with $b = 1/a = \gamma$, $d = 1$. See text for discussion.*

thus manifesting an RS effect, at time $t'_1 = \gamma\beta L/(2c)$ at an angle θ'_1 to the x'-axis, where $\tan\theta'_2 = H/[\gamma(L/2-\beta s)]$ (Fig. 4c). The flight paths of the signals from GPS2 and GPS1 in the receiver rest frame are of length s'_1, s'_2 respectively. If only the leading $\mathrm{O}(\beta)$ terms are retained in the formulas, the time dilation and length contraction effects are neglected so that the relative velocity of R and the signals is the same in the frames S and S'. If c'_1 and c'_2 are the absolute values, in the frame S', of the velocities of the signals emitted by GPS1 and GPS2 respectively, on making this approximation, then invariance of the x components of the relative velocities of R and the signals (the classical Sagnac effect) gives the relations

$$c\cos\theta - v = c'_1\cos\theta'_1, \quad c\cos\theta + v = c'_2\cos\theta'_2 \tag{6.1}$$

from which follow

$$c'_1 = \frac{(c\cos\theta - v)s'_1}{\frac{L}{2}-\beta s}, \quad c'_2 = \frac{(c\cos\theta + vc)s'_2}{\frac{L}{2}+\beta s} \tag{6.2}$$

where

$$s'_1 = \sqrt{H^2+(L/2-\beta s)^2}, \quad s'_2 = \sqrt{H^2+(L/2+\beta s)^2}$$

giving for the times-of-flight of the signals in the frame S':

$$T'_1 = \frac{s'_1}{c'_1} = \frac{L/2-\beta s}{c\cos\theta - v}, \quad T'_2 = \frac{s'_2}{c'_2} = \frac{L/2+\beta s}{c\cos\theta + v}. \tag{6.3}$$

Since, from the geometry of Fig. 4a, $\cos\theta = L/(2s)$, Eqs. (6.3) give equal times-of-flight for the signals:

$$T'_1 = T'_1 = \frac{s}{c} = T \tag{6.4}$$

where T is the time of flight of either signal in the frame S. The difference between the times of reception of the signals at R in the frame S' is then equal to the difference between the times of emission, due to the RS effect, in this frame:

$$\Delta t' \equiv t'_2 - t'_1 = \frac{\beta L}{c} + \mathrm{O}(\beta^2) \qquad \text{(RS)} \tag{6.5}$$

which may be compared with $\Delta t'_{\mathrm{RS}}$ in the experiment described in the previous section as given by Eq. (4.2). In the case that the event transformation between the frames S and S' is performed using Eqs. (3.1) and (3.2) the RS effect is absent and $\Delta t' = 0$.

As in the discussion of Fürth's proposed experiment in the previous section, if the Sagnac effect is not correctly taken into account and the speed of the signals is assumed to be the same (as predicted by the RPVAR) in the frames S and S', a time-of-flight difference equal and opposite to the RS effect $\beta L/c$ is obtained, and again $\Delta t' = 0$.

The distance parameter L in (6.5) is the separation between two adjacent satellites of a GPS constellation: $L = 3.76 \times 10^4$ km, to be compared with a typical value $L = 400$km in Eq. (4.2). With $\beta = 2.56 \times 10^{-5}$ the value of $\Delta t'$ in Eq. (6.5) is 3.3 μs or 960m at the speed of light. This may be compared with the horizontal spatial accuracy of 100 m for the Standard Positioning Service (SPS), or of 22 m for the Precision Positioning Service (PPS) [22]. As for the experiment shown in Fig.1, the expected effect is sufficiently large to be observed, with a huge statistical significance, in a single 'pass' of the experiment.

The possibility to observe such an effect, using the GPS, has been previously mentioned in Ref. [21] in a section entitled 'Confusion and consternation':

> *A 1995 meeting sponsored by the Army Research Laboratory considered the case of a rapidly moving GPS receiver. Did one, in such a case, need a coordinate system with its origin attached to the receiver in order properly to deal with clock synchronisation? From the fast-moving receiver's point of view it would seem that the GPS satellite clocks would not be synchronised. One can estimate the discrepancies from the approximate synchronisation correction vx/c^2, where v is the receiver's speed through the ECI frame and x is the distance to the GPS satellite in question. Suppose the receiver is itself in low Earth orbit (7.6 km/s) and the GPS satellite is 20 000 km ahead. Then the synchronization correction comes to 1.7 μs. That's enough time for an electromagnetic signal to travel 500 m, so one would have to correct for it.*
>
> *Within the framework of general relativity, however one coordinate system should be as legitimate as another. Measurements by an observer travelling with a moving receiver can just as well be described in another reference frame by using transformations that relate the two frames. In the special case of two inertial frames in uniform relative motion, these are the familiar Lorentz transformations.*

Indeed it is just the 'familiar Lorentz transformations' that predict the huge time difference $\Delta t'$, found in Eq. (6.5), corresponding to the ECI frame configuration shown in Fig. 3 and Fig. 4a, which is just the one discussed in the passage quoted above from Ref. [21].

The effect was incorrectly estimated in Ref. [21] by using the shortest distance between a GPS satellite and the low Earth orbit, rather than the distance L in Fig.3. However, order of magnitude of the effect was correctly given, even though no definite experimental test was proposed and there was no explanation why the very large correction predicted is, apparently, not applied in operating the GPS.

In view of the large observable effect predicted for the experiment in Fig. 3, it is interesting to consider a similar experiment where the GPS receiver is not on the satellite R, but at a fixed point on the Earth's surface, as in the usual operational mode of the GPS. In this case β in the formula $\Delta t = \beta L/c$ is not the orbital velocity of R of $\simeq 7.6$ km/sec but is rather the projection into the plane of the GPS satellite orbits of the velocity of the receiver due to the rotation of the Earth. The maximum effect occurs for a receiver at the Equator, viz: $v_{rot} \cos 55^\circ = 0.47 \times 0.574 = 0.27$ km/sec[4]. This corresponds to a value of Δt of 113 ns, or 34 m at the speed of light. As this is of the same order as the PPS accuracy, and three times smaller than the SPS accuracy, no appreciable effects due to the non-vanishing of $\Delta t'$, in the case that RS exists, is to be expected on the SPS performance at fixed points on the surface of the Earth.

[4]The planes of the orbits of GPS satellites are at an angle of 55° to the equatorial plane of the Earth [21].

7 Summary

As discussed in Ref. [16], of the three space-time 'effects' of SR: RS, LS and TD, only the latter has been, to date, experimentally verified. The RS effect predicts that two simultaneous events in a frame S', separated by a distance L in the direction of motion of another inertial frame, S, will be observed with a time separation $\delta t = \gamma\beta L/c$ in this last frame, where $v = \beta c$ is the relative velocity of the two frames.

In the first proposed experiment S' is the co-moving inertial frame of two satellites in low Earth orbit, separated by a distance of a few hundred kilometers, while S is the frame of a microwave transmitter on the surface of the Earth. The value of δt for the pair of events considered is $\simeq 30$ ns to be compared with the time resolution of a similar experiment [19] of ≤ 1 ns. This experiment is in many ways similar to that [3] proposed, also employing microwave signals, by Fürth some forty-six years ago, that is discussed in Section 5 above.

In the second proposed experiment, S is the ECI frame of the GPS satellite system, while the GPS signal receiver is in the co-moving inertial frame S' of a single satellite in low Earth orbit. The corresponding value of δt is 3.2 μs as compared to the time resolution of $\simeq 70$ ns for the PPS system [18]. Time shifts of this order of magnitude in similar experiments have been previously pointed out [21] but no corresponding test for the RS effect proposed. For both experiments, all systematic effects in time interval measurements are expected to be completely negligible in comparison with the predicted values of δt, in the case that the RS effect exists.

In another paper by the present author [16] it has been suggested, in order to avoid certain casual paradoxes of SR [15], and to ensure translational invariance, that the origin of the frame S' in the LT should be, in all cases, chosen to coincide with the position of the transformed event (a 'local' LT). This is equivalent to the use of Eqns(3.1) and (3.2) above with $b = 1/a = \gamma$ to describe the clocks. In this case it is predicted that $\delta t = 0$ in both of the experiments presented above, i.e. that there is no RS effect

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