ELSEVIER
Nuclear Instruments and Methods in Physics Research A 481 (2002) 130–143
NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
www.elsevier.com/locate/nima

Signal propagation, termination, crosstalk and losses in resistive plate chambers

Werner Riegler[a,*], Daniel Burgarth[a,1]

[a] *EP Division, CERN, CH-1211 Geneva 23, Switzerland*

Received 16 February 2001; received in revised form 27 March 2001; accepted 30 March 2001

Abstract

We discuss the signal propagation, strip termination and crosstalk in resistive plate chambers (RPCs) by analyzing the explicit time domain solution of a two dimensional multi-conductor transmission line. It is shown that all the effects can be calculated by elementary matrix manipulations. It is also shown that losses should not play a role for frequencies <200 MHz. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 29.40.Cs; 29.40.Gx

Keywords: Resistive plate chambers; RPC; RPCs; Crosstalk; Transmission lines; Reflection; Termination; Modal dispersion

1. Introduction

In many large size detectors the readout electrodes (strips) are very long i.e. the signal propagation time is large compared to the signal width. In that case the readout electrodes have to be treated as a multi-conductor transmission line. The induced signal acts as current source at some point along the electrode. In order to avoid multiple reflections, the strips have to be terminated properly on at least one end. The strip-end that is connected to the signal amplifiers has to be designed such that the crosstalk is minimized.

In this note we will discuss a RPC with a geometry similar to the design used for LHCb [1] and ATLAS [2] to illustrate a very powerful formalism for analyzing signal propagation, transmission line termination and crosstalk. The results are of course applicable to any detector geometry that satisfies the requirements for a two-dimensional transmission line. We will first introduce the general theory and then apply the formalism to some realistic geometries.

2. General solution

The theory of multi conductor transmission lines is well developed. In this chapter we list the general solutions without proof, details can be found in Ref. [3]. We assume here that the width of the readout strips is small compared to their length and that the line is uniform meaning that the geometry is independent of z (Fig. 1). In that case the detector is a two-dimensional N-conductor

*Corresponding author.
E-mail address: werner.riegler@cern.ch (W. Riegler).
[1] CERN summer student, now at Albert-Ludwigs-Universität, 79085 Freiburg, Germany.

0168-9002/02/$ - see front matter
PII: S 0 1 6 8 - 9 0 0 2 (0 1) 0 1 2 4 9 - 9

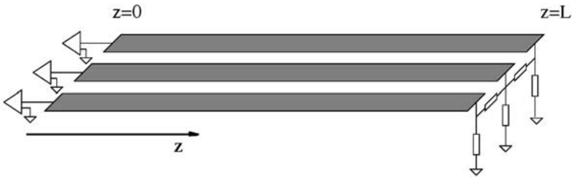

Fig. 1. Two-dimensional multi-conductor transmission line. The formalism for 2-dimensional transmission lines applies if the cross-section is independent of z. The cross-section of the individual conductors can be different.

transmission line and it is completely defined by the $N \times N$ matrices $\hat{\mathbf{C}}$, $\hat{\mathbf{L}}$, $\hat{\mathbf{R}}$ and $\hat{\mathbf{G}}$, the 'per unit length' *capacitance*, *inductance*, *resistance*, and *transconductance matrix*. For the examples given later they were calculated with Maxwell [4] a finite element field simulator program. In case these matrices are independent of frequency (which will be justified in the last chapter), the equations describing the most general two-dimensional N conductor transmission line in the TEM [3] approximation are

$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\hat{\mathbf{R}}\mathbf{I}(z,t) - \hat{\mathbf{L}}\frac{\partial}{\partial t}\mathbf{I}(z,t) \tag{1}$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\hat{\mathbf{G}}\mathbf{V}(z,t) - \hat{\mathbf{C}}\frac{\partial}{\partial t}\mathbf{V}(z,t) \tag{2}$$

where

$$\mathbf{I}(z,t) = \begin{pmatrix} I_1(z,t) \\ \cdot \\ I_N(z,t) \end{pmatrix} \qquad \mathbf{V}(z,t) = \begin{pmatrix} V_1(z,t) \\ \cdot \\ V_N(z,t) \end{pmatrix}$$

are the currents and voltages of the N individual conductors at time t and position z along the transmission line. If losses can be neglected (which will be justified in the last chapter) the matrices $\hat{\mathbf{R}}$ and $\hat{\mathbf{G}}$ are zero and the above equations simplify to

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\mathbf{I}(z,t) = \hat{\mathbf{C}}\hat{\mathbf{L}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{I}(z,t) \quad \frac{\mathrm{d}^2}{\mathrm{d}z^2}\mathbf{V}(z,t) = \hat{\mathbf{L}}\hat{\mathbf{C}}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{V}(z,t)$$

The general solution of these equations is

$$\mathbf{I}(z,t) = \hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+\left(t-\frac{z}{v_1}\right) \\ \cdot \\ I_N^+\left(t-\frac{z}{v_N}\right) \end{pmatrix} - \begin{pmatrix} I_1^-\left(t+\frac{z}{v_1}\right) \\ \cdot \\ I_N^-\left(t+\frac{z}{v_N}\right) \end{pmatrix}\right)$$

$$\mathbf{V}(z,t) = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+\left(t-\frac{z}{v_1}\right) \\ \cdot \\ I_N^+\left(t-\frac{z}{v_N}\right) \end{pmatrix} + \begin{pmatrix} I_1^-\left(t+\frac{z}{v_1}\right) \\ \cdot \\ I_N^-\left(t+\frac{z}{v_N}\right) \end{pmatrix}\right)$$

where the $I_m^+(x)$ and $I_m^-(x)$ are $2N$ arbitrary functions and

$$\hat{\mathbf{T}}^{-1}\left(\hat{\mathbf{C}}\hat{\mathbf{L}}\right)\hat{\mathbf{T}} = \hat{\mathbf{v}}^{-2} \qquad \hat{\mathbf{Z}}_C = \sqrt{\hat{\mathbf{L}}/\hat{\mathbf{C}}} = \hat{\mathbf{L}}\hat{\mathbf{T}}\hat{\mathbf{v}}\hat{\mathbf{T}}^{-1} \tag{3}$$

with

$$\hat{\mathbf{v}}^{-2} = \begin{pmatrix} \frac{1}{v_1^2} & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & \frac{1}{v_N^2} \end{pmatrix} \qquad \hat{\mathbf{v}} = \begin{pmatrix} v_1 & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & v_N \end{pmatrix}$$

The matrix $\hat{\mathbf{T}}$ contains the normalized eigenvectors of the matrix $\hat{\mathbf{C}}\hat{\mathbf{L}}$ and $1/v_i^2$ are the corresponding eigenvalues. The matrix $\hat{\mathbf{Z}}_C$ is called the *characteristic impedance matrix*. The individual functions represent pulses that are running along the strips in positive and negative direction without changing their shape. Note that in general the signal propagation happens with N different velocities and also note that the solution is completely general.

The explicit form of these functions is determined by the line excitation mechanism and boundary conditions at the strips ends $z = 0$ and $z = L$. A detector signal acts as an ideal current source $I^0(t)$ at a position $z = z_0$ somewhere along a conductor n, which defines the $2N$ functions.

Hence we have the general solution

$$\mathbf{I}(z,t) = \frac{1}{2}\hat{\mathbf{T}}\left(\begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z-z_0}{v_N}\right)\end{pmatrix} - \begin{pmatrix} t_{1n}^{-1} I^0\left(t + \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t + \frac{z-z_0}{v_N}\right)\end{pmatrix}\right) \quad (4)$$

$$\mathbf{V}(z,t) = \frac{1}{2}\hat{\mathbf{Z}}_C\hat{\mathbf{T}}\left(\begin{pmatrix} t_{1n}^{-1} I^0\left(t - \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t - \frac{z-z_0}{v_N}\right)\end{pmatrix} + \begin{pmatrix} t_{1n}^{-1} I^0\left(t + \frac{z-z_0}{v_1}\right) \\ \cdot \\ t_{Nn}^{-1} I^0\left(t + \frac{z-z_0}{v_N}\right)\end{pmatrix}\right) \quad (5)$$

which we can write as

$$\mathbf{I}(z,t) = \mathbf{I}^+(z,t) - \mathbf{I}^-(z,t)$$

$$\mathbf{V}(z,t) = \hat{\mathbf{Z}}_C\left[\mathbf{I}^+(z,t) + \mathbf{I}^-(z,t)\right] = \mathbf{V}^+(z,t) + \mathbf{V}^-(z,t).$$

The t_{nm}^{-1} are the elements of the matrix $\hat{\mathbf{T}}^{-1}$. It is easy to see that at $z = z_0$ it holds that $\mathbf{I}^+(z_0,t) + \mathbf{I}^-(z_0,t) = \left(0,.,I^0(t),.,0\right)^{\mathrm{T}}$, so it satisfies the required boundary condition. This solution shows that there are pulses running symmetrically in the positive and negative direction from the point z_0. The pulse running along one conductor is a superposition of N times the same pulse-shape $I^0(t)$ running with N different velocities v_i. Therefore we find signal dispersion even for a lossless transmission line which is called *modal dispersion*.

The pulses will travel until they hit the strip ends where they are reflected according to the connected networks. We assume now an arbitrary interconnection of strips at $z = 0$ and $z = L$ with purely resistive loads. For $z = L$ we define R_{ij} $i \neq j$ the resistors between strip i and j and R_{ii} the resistors between strip i and ground. The boundary condition is then given by

$$\mathbf{V}(L,t) = \hat{\mathbf{Z}}_T\mathbf{I}(L,t) \qquad \hat{\mathbf{Z}}_T = \hat{\mathbf{Y}}_T^{-1}$$

$$Y_{ij}^{\mathrm{T}} = -\frac{1}{R_{ij}} \quad i \neq j \qquad Y_{ii}^{\mathrm{T}} = \sum_{j=1}^{N}\frac{1}{R_{ij}} \quad (6)$$

where we define $\hat{\mathbf{Z}}_T$ as the *load impedance matrix*. The other strip end at $z = 0$ will of course be characterized by a different load impedance matrix which we call $\hat{\mathbf{Z}}_P$ since we assume it is the readout (preamplifier) side. The effect of the boundary is that the voltage pulses are reflected according to

$$\mathbf{V}_{\mathrm{refl}}^- = \hat{\boldsymbol{\Gamma}}_T\mathbf{V}^+ \quad \text{and} \quad \mathbf{V}_{\mathrm{refl}}^+ = \hat{\boldsymbol{\Gamma}}_P\mathbf{V}^-$$

where the *reflection coefficient matrix* Γ at the line ends is defined as

$$\hat{\boldsymbol{\Gamma}}_T = \left(\hat{\mathbf{Z}}_T - \hat{\mathbf{Z}}_C\right)\left(\hat{\mathbf{Z}}_T + \hat{\mathbf{Z}}_C\right)^{-1}$$

$$\hat{\boldsymbol{\Gamma}}_P = \left(\hat{\mathbf{Z}}_P - \hat{\mathbf{Z}}_C\right)\left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1}$$

and the actual voltages at the strip ends are given by

$$\mathbf{V}(L,t) = \mathbf{V}^+ + \mathbf{V}_{\mathrm{refl}}^- = \left(\hat{\mathbf{1}} + \hat{\boldsymbol{\Gamma}}_T\right)\mathbf{V}^+ \quad (7)$$

$$\mathbf{V}(0,t) = \mathbf{V}^- + \mathbf{V}_{\mathrm{refl}}^+ = \left(\hat{\mathbf{1}} + \hat{\boldsymbol{\Gamma}}_P\right)\mathbf{V}^-. \quad (8)$$

The matrix $\hat{\mathbf{1}} = \mathrm{Diag}(1, \ldots, 1)$ is the unity matrix. This is our final solution. Given the current pulse $I^0(t)$ at position $z = z_0$ on conductor n we know the two pulses $\mathbf{V}^+$ and $\mathbf{V}^-$ running symmetrically in both directions from $z = z_0$ towards the two line ends from Eq. (5). The networks at the line ends define the matrices $\hat{\boldsymbol{\Gamma}}_T$ and $\hat{\boldsymbol{\Gamma}}_P$ which give the reflected and measured pulses. If the transmission line is not terminated we of course have to add up the multiple reflections.

3. Termination

If we want to eliminate reflections at the line end $z = L$ the reflection coefficient matrix Γ_T has to be zero, i.e. the load impedance matrix $\hat{\mathbf{Z}}_T$ has to be equal to the characteristic impedance matrix $\hat{\mathbf{Z}}_C$. The termination resistors R^{T}_{ij} are calculated by inverting Eq. (6) giving

$$\hat{\mathbf{Y}}_C = \hat{\mathbf{Z}}_C^{-1} \quad R^{\mathrm{T}}_{ij} = -\frac{1}{Y^C_{ij}} \quad i \neq j \quad \frac{1}{R^{\mathrm{T}}_{ii}} = \sum_{j=0}^{N} Y^C_{ij}. \tag{9}$$

We see that in order to eliminate reflections we theoretically have to interconnect all the conductors i.e. we need $\frac{1}{2}N(N+1)$ termination resistors. Examples will be discussed later.

4. Measured signal

Now we assume that one end ($z = L$) of the transmission line is perfectly terminated. The other end ($z = 0$) is read by preamplifiers and is loaded by $\mathbf{Z}_P$. If the current $I^0(t)$ is induced on strip n, the voltage and current measured by the amplifiers is calculated from Eqs. (5) and (8) which gives

$$\mathbf{V}_{\mathrm{meas}}(t) = \mathbf{V}(0, t)$$

$$= \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \hat{\mathbf{T}} \begin{pmatrix} t^{-1}_{1n} I^0\left(t - \frac{z_0}{v_1}\right) \\ \cdot \\ t^{-1}_{Nn} I^0\left(t - \frac{z_0}{v_N}\right) \end{pmatrix} \tag{10}$$

and

$$I_{\mathrm{meas}}(t) = \frac{1}{R_{\mathrm{in}}} \mathbf{V}_{\mathrm{meas}}(t) \neq \mathbf{I}(0, t) \tag{11}$$

R_{in} is the preamplifier input resistance. $\mathbf{I}(0,\ t)$ is the current at the line end which is different from the current flowing through the amplifier in case the strips are interconnected on the amplifier side. The relative amplitudes of the voltages give the crosstalk which we discuss next.

5. Crosstalk

The above solution allows us to write down the explicit formula for the crosstalk from the signal strip n to all other strips.

5.1. Homogeneous and inhomogeneous transmission lines

In case the volume, where the electro-magnetic waves propagate, has uniform dielectric properties, all the propagation velocities are the same and we call the geometry a *homogeneous* transmission line. It then holds that

$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2}\hat{\mathbf{1}} \qquad \hat{\mathbf{Z}}_C = v\hat{\mathbf{L}}$$

An example is the geometry shown in Fig. 3. The RPC geometry that we want to study (Fig. 9) is however an *inhomogeneous* transmission line and we will therefore find N different propagation velocities causing signal dispersion even if the transmission line is lossless.

5.2. Transmission line with small dispersion

If all the propagation velocities are the same or if the transmission line is short, such that the dispersion is very small, the solution from Eq. (10) evaluates to

$$\mathbf{V}_{\mathrm{meas}}(t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} \times \left(0, ., 0, I^0\left(t - \frac{z_0}{v}\right), 0, ., 0\right)^{\mathrm{T}} \tag{12}$$

Defining the Matrix $\hat{\mathbf{M}} = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1}$ the crosstalk from conductor n to conductor m is given by $V_m/V_n = M_{mn}/M_{nn}$. If we adjust the preamplifier input resistance and interconnecting resistors such that $\hat{\mathbf{Z}}_P = \hat{\mathbf{Z}}_C$ i.e. if we ideally terminate the preamplifier side, the solution becomes

$$\mathbf{V}_{\mathrm{meas}}(t) = \frac{1}{2}\hat{\mathbf{Z}}_C \left(0, ., 0, I^0\left(t - \frac{z_0}{v}\right), 0, ., 0\right)^{\mathrm{T}}$$

The crosstalk from signal strip n to strip m is then given by Z^C_{nm}/Z^C_{nn}. We see that in order to have small crosstalk, the off-diagonal elements in the characteristic impedance matrix $\hat{\mathbf{Z}}_C$ should be small compared to the diagonal elements. Terminating the preamplifier side is however *not* the optimum scenario in terms of collected charge and crosstalk which will be shown next.

If we do not interconnect the strips on the preamplifier side but just connect each strip to the preamplifier, it holds that $\hat{\mathbf{Z}}_P = \mathrm{Diag}(R_{\mathrm{in}}, R_{\mathrm{in}}, \ldots, R_{\mathrm{in}})$ where R_{in} is the preamplifier input resistance. In case the preamplifier input resistance is $R_{\mathrm{in}} = 0$ we have $\hat{\mathbf{Z}}_P = 0$, $\mathbf{V}_{\mathrm{meas}}(t) = 0$ and

$$I_{\mathrm{meas}}(t) = \left(0, 0, \ldots, I^0\left(t - \frac{z_0}{v}\right), \ldots, 0\right)^{\mathrm{T}}$$

i.e. we measure exactly the pulse induced on line n and zero on all the other lines. The whole process looks the following: if a current pulse is induced at point $z = z_0$, half of it runs to the left and half of it to the right. The pulse running to the right is absorbed in the termination network ($z = L$), the pulse running to the left is totally negatively reflected ($z = 0$) and the preamplifiers measure the difference i.e. the entire current signal. The reflection again runs to the right where it is absorbed. This way we measure the maximum signal with minimum crosstalk. It is evident that interconnecting the strips on the preamplifier side i.e. introducing off-diagonal elements in the load impedance matrix $\hat{\mathbf{Z}}_P$, will always increase the crosstalk.

Therefore we conclude for a terminated transmission line with small dispersion that the measured signals on all strips have the same shape as the original induced signal, the crosstalk is independent of the position of the induced signal along the strip, the signal will be maximal and the crosstalk minimal if we do not interconnect the strips and the preamplifier input resistance R_{in} is zero (or lowest possible).

5.3. Transmission line with significant dispersion

For a long, inhomogeneous transmission line the individual pulses will disperse as they run along the strips and the pulse-shapes will change. The crosstalk will therefore in general increase as a function of distance from the preamplifier and will also depend on the shape of the induced signal. If we integrate the current flowing through the preamplifier (Eq. (11)) for sufficient time, the N expressions $\int I\left(t - (z - z_0)/v_n\right)\,\mathrm{d}t$ evaluate to the same value $q = \int I(t)\,\mathrm{d}t$ since the pulses are just time shifted but have the same shape. Therefore we find

$$\mathbf{Q} = \int \mathbf{I}_{\mathrm{meas}}(t)\,\mathrm{d}t$$

$$= \frac{1}{R_{\mathrm{in}}} \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C \left(\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C\right)^{-1} (0, ., 0, q, 0, ., 0)^{\mathrm{T}} \quad (13)$$

where q is the total charge *induced* on strip n, and the Q_m are the charges *measured* on the N strips. The charge fraction measured on the neighbouring strips ('crosstalk charge') is given as before by $Q_m/Q_n = M_{mn}/M_{nn}$. This will be the observed crosstalk for 'slow' readout electronics i.e. preamplifiers with an integration time that is much larger than the signal dispersion time. Fast amplifiers will however sense the signal dispersion and will therefore show more crosstalk. In case $R_{\mathrm{in}} = 0$ and in case the strips are not interconnected on the amplifier side the above expression becomes

$$\mathbf{Q} = (0, ., 0, q, 0, ., 0)^{\mathrm{T}}. \quad (14)$$

The crosstalk charge is zero which means that all the crosstalk signals are perfectly bipolar.

We conclude on the transmission line with significant dispersion that the signal shapes change as a function of distance from the preamplifier and are only equal to the original induced signal if the current is induced close to the preamplifier. Therefore, the amplitude and shape of the crosstalk signal also changes as a function of distance from the amplifier. In general the crosstalk will increase as a function of distance from the amplifier. The crosstalk is lowest if the strips are not interconnected and the amplifier input resistance is as low as possible. The crosstalk is smaller for slow electronics.

6. Examples

In this section we apply the formalism to an actual RPC geometry. To study all aspects we discuss a single strip RPC, a homogeneous double strip transmission line, a double strip RPC and finally a RPC with many strips and guard strip.

6.1. Single strip RPC

A single strip RPC is shown in Fig. 2. The parameters calculated with Maxwell are

$$C = 205\,\text{pF/m} \quad L = 89.3\,\text{nH/m} \Rightarrow Z_C = 20.87\,\Omega$$

$$v = 2.34\times10^8\,\text{m/s}$$

The strip is terminated simply by putting a termination resistor $R_T = Z_C$ at the strip end $z = L$. The signal measured by an amplifier at $z = 0$ with input resistance R_{in} is given by

$$I_{\text{meas}}(t) = \frac{Z_C}{Z_C + R_{\text{in}}} I^0\left(t - \frac{z_0}{v}\right)$$

so it has the same shape as the original induced signal, independent of z_0. In case the preamplifier input impedance is $R_{\text{in}} = 0$ the measured signal is equal to the induced signal.

6.2. Homogeneous double strip line

To see the difference between an homogeneous and inhomogeneous transmission line we study the above RPC geometry with two strips and first omit the Bakelite (Fig. 3). In that case the dielectric properties are equal in the entire area where the waves propagate and the propagation velocity will be equal for all waves.

The Maxwell calculations give

$$\hat{\mathbf{C}} = \begin{pmatrix} 126 & -6.4 \\ -6.4 & 126 \end{pmatrix} \text{pF/m}$$

$$\hat{\mathbf{L}} = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \text{nH/m}$$

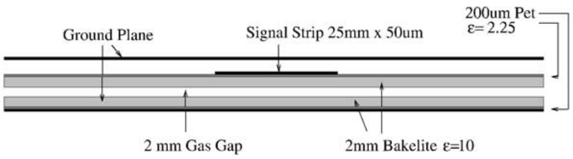

Fig. 2. Cross-section through a RPC with a single signal strip. The current signal is induced by the avalanche electrons moving in the gas gap.

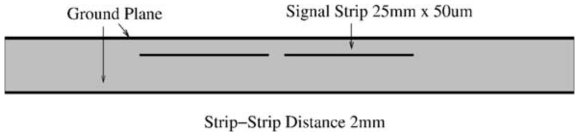

Fig. 3. Cross-section through a homogeneous transmission line with two strips.

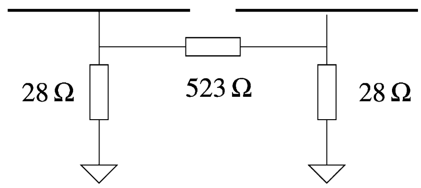

Fig. 4. Termination network for a double strip RPC calculated from Eq. (9). In order to perfectly terminate a multi-conductor transmission line one has to interconnect all the lines.

from which we calculate with Eqs. (3) and (9)

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 26.5 & 1.34 \\ 1.34 & 26.5 \end{pmatrix} \Omega$$

$$\hat{\mathbf{R}}_T = \begin{pmatrix} 27.8 & 522.7 \\ 522.7 & 27.8 \end{pmatrix} \Omega \quad v = 3\times10^8\,\text{m/s}$$

The capacitance matrix $\hat{\mathbf{C}}$ is defined such that the negative off-diagonal element $-C_{ij}$ is the mutual capacitance between conductor i and j, and the *sum* of the ith column $\sum_j C_{ij}$ is the capacitance of conductor i to ground. Since the matrix $\hat{\mathbf{C}}\hat{\mathbf{L}}$ is already diagonal the matrix $\hat{\mathbf{T}}$ is undefined and any set of two orthonormal vectors will do for it. The matrix $\hat{\mathbf{R}}_T$ contains the termination resistors calculated from Eq. (9). Only if we interconnect the strips on the termination side we avoid reflections (Fig. 4). The other side of the strips we finally want to read out with amplifiers of input resistance R_{in}. We do not interconnect the strips on this side and the load impedance matrix is a diagonal matrix with R_{in} as diagonal elements. The measured current for an induced current pulse $I^0(t)$ at $z = z_0$ of strip 1 is given by Eq. (11) and evaluates to

$$\begin{pmatrix} I^1_{\rm meas}(t) \\ I^2_{\rm meas}(t) \end{pmatrix} = \frac{1}{R_{\rm in}^2 + 2R_{\rm in}Z_{11} + Z_{11}^2 - Z_{12}^2} \times \begin{pmatrix} R_{\rm in}Z_{11} + Z_{11}^2 - Z_{12}^2 \\ R_{\rm in}Z_{12} \end{pmatrix} I^0\left(t - \frac{z_0}{c}\right)$$

so we find a crosstalk of

$$\frac{I^2_{\rm meas}(t)}{I^1_{\rm meas}(t)} = \frac{R_{\rm in}Z_{12}}{R_{\rm in}Z_{11} + Z_{11}^2 - Z_{12}^2} \qquad (15)$$

which is illustrated in Fig. 5. The measured signals on both strips have exactly the same shape as the original induced signal $I^0(t)$. The crosstalk is zero if the preamplifier input resistance is zero. In that case we measure exactly the induced current signal. In order to keep the crosstalk small we want the ratio Z_{12}/Z_{11} to be small, so the off-diagonal elements in the impedance matrix should be small compared to the diagonal ones. In the limit of $R_{\rm in} \to \infty$ the crosstalk goes to Z_{12}/Z_{11} and the pulse-height goes to zero.

6.3. Double strip RPC

Adding the Bakelite to the geometry discussed above (Fig. 6), Maxwell gives the characteristic parameters

$$\hat{\mathbf{C}} = \begin{pmatrix} 216 & -30 \\ -30 & 216 \end{pmatrix} \text{pF/cm}$$

$$\hat{\mathbf{L}} = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \text{nH/m}$$

from which we find with Eqs. (3) and (9)

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} 20.4 & 1.93 \\ 1.93 & 20.4 \end{pmatrix} \Omega \quad \hat{\mathbf{T}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\hat{\mathbf{R}}_T = \begin{pmatrix} 22.3 & 213.7 \\ 213.7 & 22.3 \end{pmatrix} \Omega$$

$$\hat{\mathbf{v}} = \begin{pmatrix} 2.2 & 0 \\ 0 & 2.4 \end{pmatrix} \times 10^8 \text{ m/s}$$

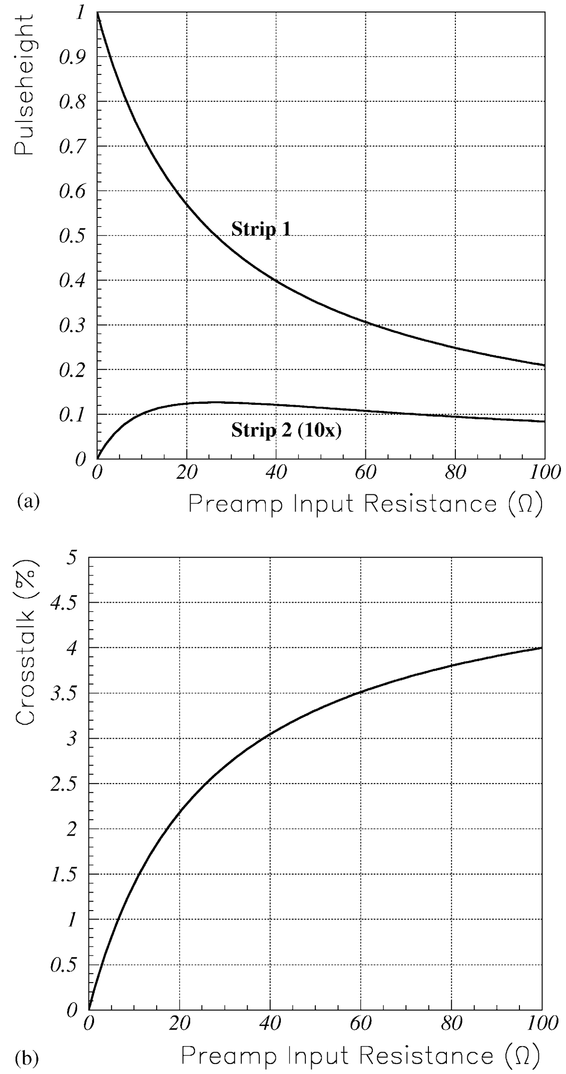

Fig. 5. (a) Pulse-height on both strips. The current is induced on strip 1. The crosstalk pulse on strip 2 is multiplied by 10 for illustration. (b) Fraction of crosstalk. We see that the pulse-height decreases and the crosstalk increases for larger preamplifier input resistance.

The Bakelite has increased the mutual strip–strip capacitance from 6.4 to 30 pF/m but has left the inductance unchanged (as expected). Therefore the off-diagonal elements in the impedance matrix are larger which will increase the crosstalk. The propagation velocities of the two modes differ by 10%. For a signal $I^0(t)$ at $z = z_0$ the preamplifiers

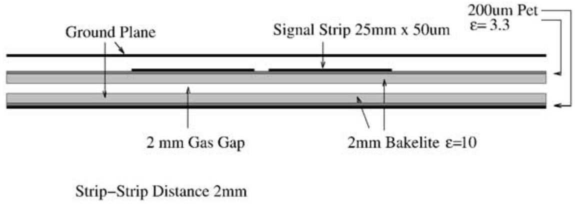

Fig. 6. RPC with two strips. The dielectric properties are *not* uniform in the area where the waves propagate which leads to different propagation velocities.

measure a current of

$$\begin{pmatrix} I^1_{\text{meas}}(t) \\ I^2_{\text{meas}}(t) \end{pmatrix} = \frac{1}{2} c_1 \times \begin{pmatrix} R_{\text{in}} Z_{11} + Z_{11}^2 - Z_{12}^2 & R_{\text{in}} Z_{12} \\ R_{\text{in}} Z_{12} & R_{\text{in}} Z_{11} + Z_{11}^2 - Z_{12}^2 \end{pmatrix} \times \begin{pmatrix} I^0\left(t - \frac{z_0}{v_1}\right) + I^0\left(t - \frac{z_0}{v_2}\right) \\ -I^0\left(t - \frac{z_0}{v_1}\right) + I^0\left(t - \frac{z_0}{v_2}\right) \end{pmatrix}$$

$$c_1 = \frac{1}{R_{\text{in}}^2 + 2 R_{\text{in}} Z_{11} + Z_{11}^2 - Z_{12}^2}$$

We see that the measured signal is a superposition (according to the matrix $\hat{\mathbf{T}}$) of two pulses with the same shape running with two different velocities.

Due to the dispersion the crosstalk will depend on the pulse-shape $I^0(t)$ and the amplifier response. We assume a current pulse-shape of

$$I^0(t) = \frac{E}{V} e_0 v_e N_0 \left(1 - \frac{t v_e}{d}\right) e^{\alpha v_e t} \quad 0 < t < d/v_e$$

where v_e is the electron drift-velocity, α is the Townsend coefficient, d is the thickness of the gas gap, N_0 is the number of primary electrons that are uniformly distributed along the track and E is the electric field in the gas gap if the signal strip is put on a voltage V. For the following calculations we assume $v_e = 100\ \mu\text{m/ns}$, $\alpha = 100\ \text{cm}^{-1}$ and $d = 2\ \text{mm}$. The dispersion effect is illustrated in Fig. 7.

If the strips are short or the signal is induced close to the preamplifier side we can neglect the different propagation times and the crosstalk is again given by Eq. (15). In that case the crosstalk

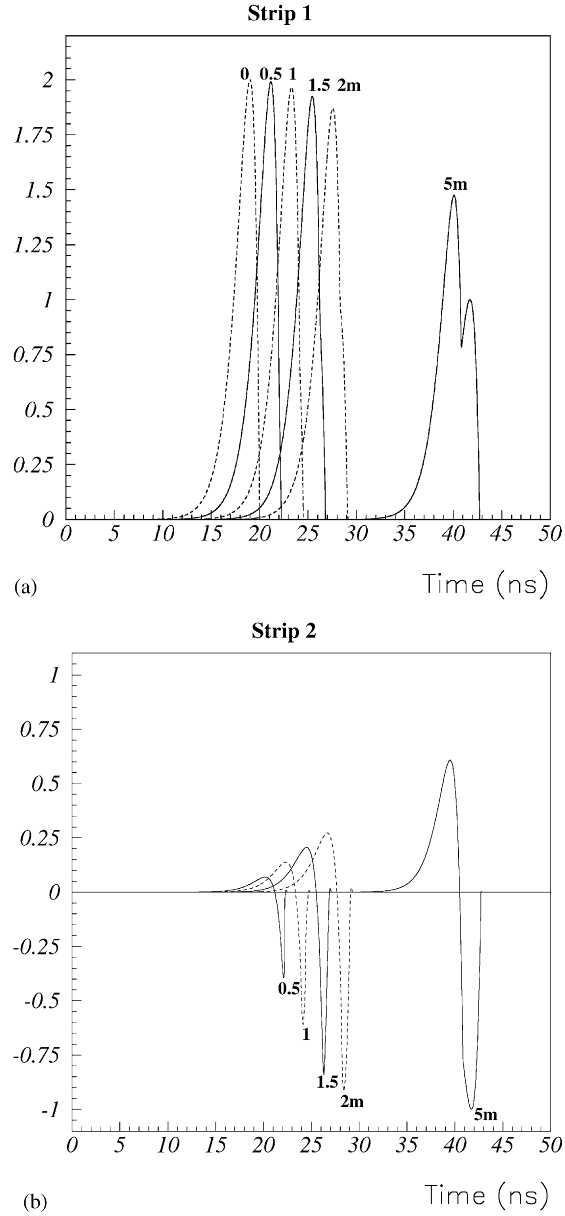

Fig. 7. Current pulses running along the two strips of the RPC shown in Fig. 6. The labels show the distance of the induced signal from the amplifier side. (a) Signal on the strip where the current is induced. One can see that the two 'modes' are dispersing after some distance. (b) Signal travelling on the neighbouring strip. At the position where the signal is induced the crosstalk signal is zero, as the modes are dispersing the crosstalk increases.

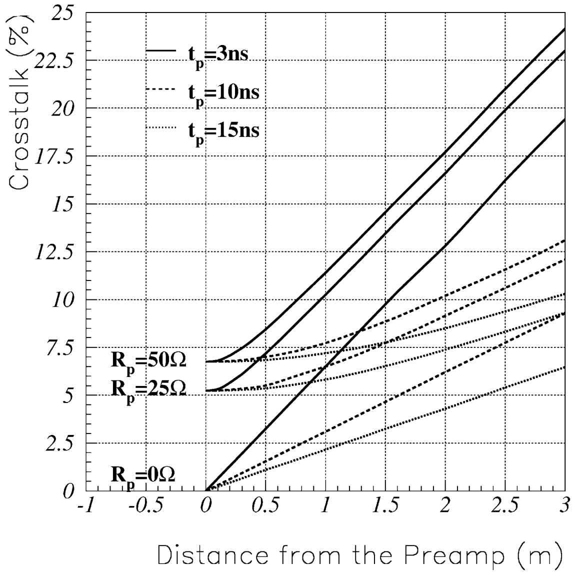

Fig. 8. Crosstalk for different preamplifier input resistances and peaking times as a function of distance from the amplifier side (RPC shown in Fig. 6). For small distances the pulses do not disperse and the crosstalk is given by Eq. (15), independent of the amplifier speed. For fast amplifiers the crosstalk increases strongly with the distance. In the limit of very long peaking times the crosstalk would become independent of the position.

is independent of the peaking time and the crosstalk signal has the same shape as the actual induced signal.

For long strips the crosstalk will depend on the amplifier response which we assume as

$$f(t) = \left(\frac{nt}{t_{\mathrm{p}}}\right)^{n} \mathrm{e}^{-\frac{nt}{t_{\mathrm{p}}}}$$

where t_{p} is the *peaking time* of the amplifier and n is the number of integration stages. In the following we will assume $n = 3$. The measured signal is given by the convolution of the current signal with the delta response $f(t)$. The result is illustrated in Fig. 8. We find a very strong dependence of the crosstalk on the amplifier peaking time and the distance.

6.4. RPC with many strips and guard strip

Finally we investigate the crosstalk for an RPC with many strips and an additional guard strip in

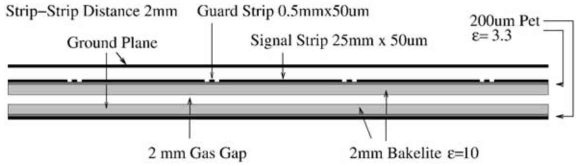

Fig. 9. RPC geometry with many strips and a guard strip to reduce the strip–strip coupling.

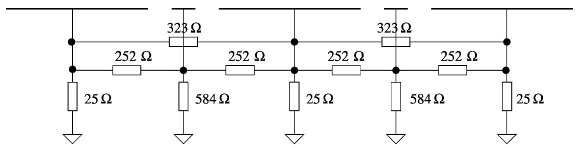

Fig. 10. Ideal termination for the RPC with many strips calculated from Eq. (9). All other interconnections are $>25\,\mathrm{k}\Omega$ and have negligible effect.

between the signal strips (Fig. 9). Due to the guard strip the cross capacitance between two signal strips reduces from 30 to 21.7 pF/m. The ideal termination network calculated from Eq. (9) is shown in Fig. 10. All other interconnections are $>25\,\mathrm{k}\Omega$ and can be neglected. It is important that the guard strip is not grounded but also included in the termination network on the 'far' side if we want to avoid any reflections. To illustrate the effect Fig. 11 shows the case where the signal strips are connected to ground with $25\,\Omega$, the strips are not interconnected and the guard strip is grounded on both sides (preamplifier and termination side). As expected we find reflections. The significance of the reflections and the question about how many termination interconnections in Fig. 10 are therefore necessary has of course to be decided for the actual application.

Connecting the strips to amplifiers with input resistance R_{in} and grounding the guard strips gives the load impendance matrix $\hat{\mathbf{Z}}_P = \mathrm{Diag}(\ldots, R_{\mathrm{in}}, 0, R_{\mathrm{in}}, 0, R_{\mathrm{in}} \ldots)$. The crosstalk versus distance to the first and second neighbour is shown in Fig. 12. The guard strip reduces the crosstalk by about 40% for signals induced close to the amplifier. Since the increase of crosstalk with distance from the amplifier is however very similar to the RPC with no guard strip, the crosstalk

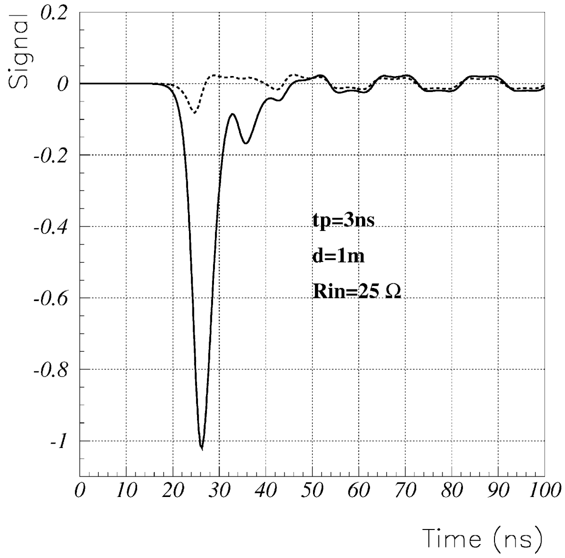

Fig. 11. Signal (solid line) and crosstalk (dashed line) for the scenario where the RPC strips are terminated at 25 Ω and the intermediate strips are grounded on both sides. As expected we find reflections.

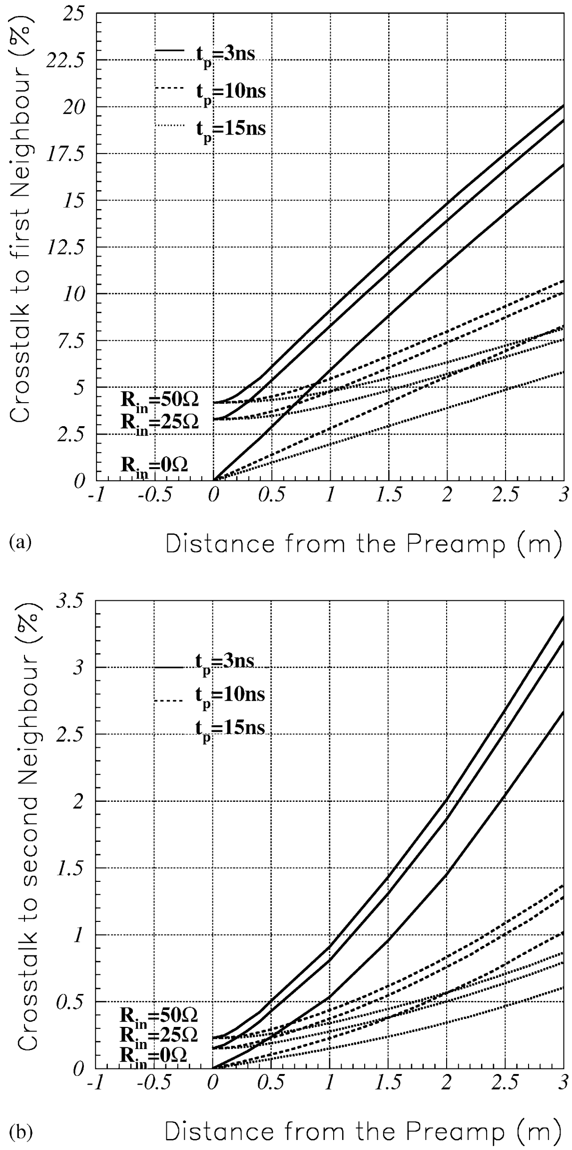

Fig. 12. Crosstalk to the first neighbour (a) and second neighbour (b) for different peaking times and preamplifier input resistances. The crosstalk to the second neighbour happens mainly through the first neighbour and not through direct coupling which can be seen by the fact that the values in the second plot are approximately the square of the first one.

numbers for large distances are very similar. The mutual capacitance to the second neighbour is very small, i.e. the crosstalk to the second neighbour happens mainly through the first neighbour. Fig. 13 shows the crosstalk signal for two different distances of the induced signal from the preamplifier side. As discussed before the shape of the crosstalk signal changes as a function of distance.

7. Frequency dependence and losses

For all the previous studies we neglected losses and assumed that $\hat{\mathbf{L}}$ and $\hat{\mathbf{C}}$ are independent of frequency. A frequency dependence of these two matrices will introduce dispersion in addition to the *modal dispersion* effect discussed earlier. For conductors with small losses $\hat{\mathbf{L}}$ will be frequency independent. The matrix $\hat{\mathbf{C}}$ however will be affected by a frequency dependence of the permittivity ε of the surrounding medium. Most dielectrics are reported to show no frequency and loss effects up to the GHz range and since preamplifiers used for RPCs rarely exceed a bandwidth of 200 MHz we should not have to worry about these effects. Bakelite however is a very bad material in that repect and shows losses

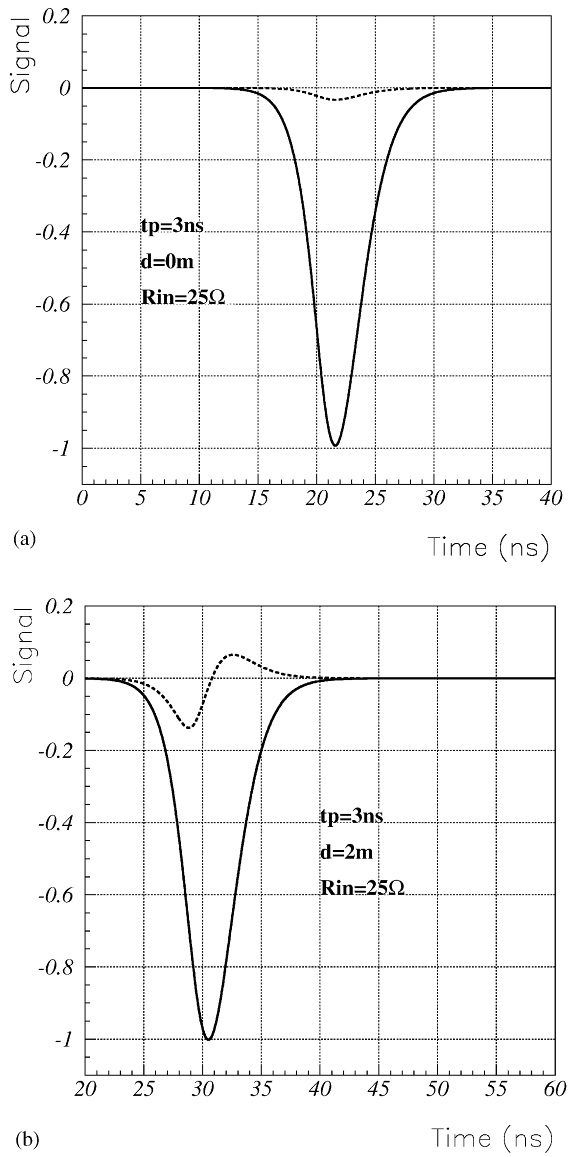

Fig. 13. Signal and crosstalk to the first neighbour strip for two different distances of the induced current from the amplifier ($d = 0$ m, 2 m). The solid line shows the signal strip, the dotted line the crosstalk. (a) Close to the amplifier the dispersion is small and the crosstalk signal has the same shape as the original one. (b) For larger distances the shape changes and the crosstalk increases. The integral over the crosstalk signal does not change as a function of the distance from the preamplifier (Eq. (13)) and therefore the crosstalk signal will become more bipolar for larger distances.

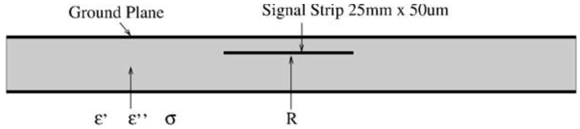

Fig. 14. Homogeneous transmission line with a single lossy conductor and a lossy surrounding medium.

and frequency effects already far below 1 GHz, so we have to check our assumptions carefully.

Two kinds of losses can occur in the given transmission lines. Losses in the conductors that will be represented by the matrix $\hat{\mathbf{R}}$ in Eq. (1) and losses in the surrounding medium (e.g. the Bakelite) which are represented by the matrix $\hat{\mathbf{G}}$ in Eq. (2). In general these losses will cause frequency dependent dispersion and exponential attenuation. A general formalism for lossy multi-conductor transmission lines exists, in this report we will however only discuss the losses for a homogeneous single conductor transmission line (Fig. 14) to estimate the effects. The losses introduce a frequency dependence, so we have to work in the frequency domain. Putting a sine-wave with amplitude A_0 on the conductor at $z = 0$ we find an attenuated and phase shifted sine-wave at position z according to

$$A(z,t) = A_0 \mathrm{e}^{-(\alpha + \mathrm{i}\beta)z} \mathrm{e}^{\omega t}$$

$$\alpha + \mathrm{i}\beta = \sqrt{(R + \mathrm{i}\omega L)(G + \mathrm{i}\omega C)} \quad (16)$$

where α is the *attenuation factor* that we are interested in. For small losses i.e. $R \ll \omega L$ and $G \ll \omega C$ we can expand the above expression and find

$$\alpha \approx \frac{1}{2}\frac{R}{Z_C} + \frac{1}{2} G Z_C \quad Z_C = \sqrt{\frac{L}{C}} \quad (17)$$

where Z_C is the *characteristic impedance* in the limit of high frequencies. The attenuation length l_{att} is then given by $1/\alpha$.

7.1. Losses due to R

The losses due to the resistance of the readout electrodes are given by the DC resistance at low frequencies and by the skin effect at high frequencies. Assuming that all the current is

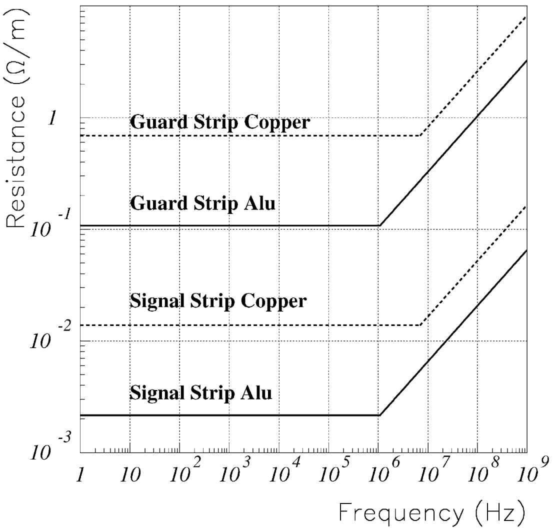

Fig. 15. Conductor resistance assuming all the current to flow within one skin-depth for the RPC geometry in Fig. 9.

flowing within one skin-depth of the conductor we find the resistance numbers given in Fig. 15 for the RPC geometry in Fig. 9 [3]. At a frequency of 1 GHz the resistance of signal and guard strip (copper) is about 0.2 and 10 Ω/m. The characteristic impedance of the RPC strips in Fig. 9 is about 20 and 120 Ω so we find attenuation lengths of 200 and 24 m for signal and guard strip which are certainly negligible for RPCs of a few metres length.

7.2. Losses due to G

The losses to the surrounding medium are due to conduction losses and polarization losses. Conduction losses due to free charge in the dielectric medium are characterized by σ, polarization losses due to bound charge in the dielectric are characterized by an imaginary permittivity ε_i. They can be included in the calculation by introducing a complex permittivity

$$\varepsilon = \varepsilon_r - \mathrm{i}\left[\varepsilon_i + \frac{\sigma}{\omega}\right]$$

Calculating the complex capacitance (or capacitance matrix) $\hat{\mathbf{C}}_I$ for this complex permittivity, the capacitance matrix $\hat{\mathbf{C}}$ and conductance matrix $\hat{\mathbf{G}}$ are given by

$$\hat{\mathbf{C}} = \mathrm{Re}\left[\hat{\mathbf{C}}_I\right] \quad \hat{\mathbf{G}} = -\omega \mathrm{Im}\left[\hat{\mathbf{C}}_I\right] \tag{18}$$

so for a homogeneous single conductor transmission line like in Fig. 14 we have

$$G = G_{\mathrm{cond}} + G_{\mathrm{pol}} = \frac{\sigma}{\varepsilon_r}C + \omega\frac{\varepsilon_i}{\varepsilon_r}C = \frac{\sigma}{\varepsilon_r}\frac{1}{vZ_C} + \omega\frac{\varepsilon_i}{\varepsilon_r}C$$

The effect from G_{cond} can best be estimated by inserting it into Eq. (17) ($R = 0$) which gives

$$\alpha \approx \frac{\sigma}{2\varepsilon_r v}$$

The Bakelite used for RPCs usually has a conductivity of $\sigma < 10^{-8}$ S/m so for a line with a permittivity of $\varepsilon_r = \varepsilon_0$ and $v = c$ we find an attenuation length $l_{\mathrm{att}} > 5 \times 10^5$ m. Therefore the effect from the conductivity σ can be completely neglected.

The effect from polarization loss G_{pol} can best be estimated by rewriting the expression in Eq. (16) as

$$\alpha + \mathrm{i}\beta = \sqrt{(R + \mathrm{i}\omega L)(G_{\mathrm{pol}} + \mathrm{i}\omega C)} = \sqrt{(R + \mathrm{i}\omega L)\left(\frac{\varepsilon_i}{\varepsilon_r} + \mathrm{i}\right)\omega C}$$

The ratio $\varepsilon_i/\varepsilon_r$ is often referred to as *dissipation factor* or *loss tangent* $\tan\,\delta_\varepsilon$. As long as the loss tangent is much smaller than unity the polarization losses can be neglected. The loss angle of Bakelite varies significantly with frequency and is also different for different kinds of Bakelite. In general the loss tangent is <0.001 below 1 GHz for most dielectric materials, but as discussed before one has to be careful with Bakelite.

For all our previous studies we only assumed $\varepsilon_r = 10$ and ε_i, $\sigma = 0$. A comparison of this model with measurements on an actual RPC is shown in Fig. 16. A voltage sine wave was connected to one strip and the amplitude on the first and second neighbour was measured. The measurement errors were estimated by checking the sensitivity of the measurement results to external variations (changing the orientation of the RPC, grounding, etc.). The simulations where performed by feeding the capacitance and inductance matrices (calculated

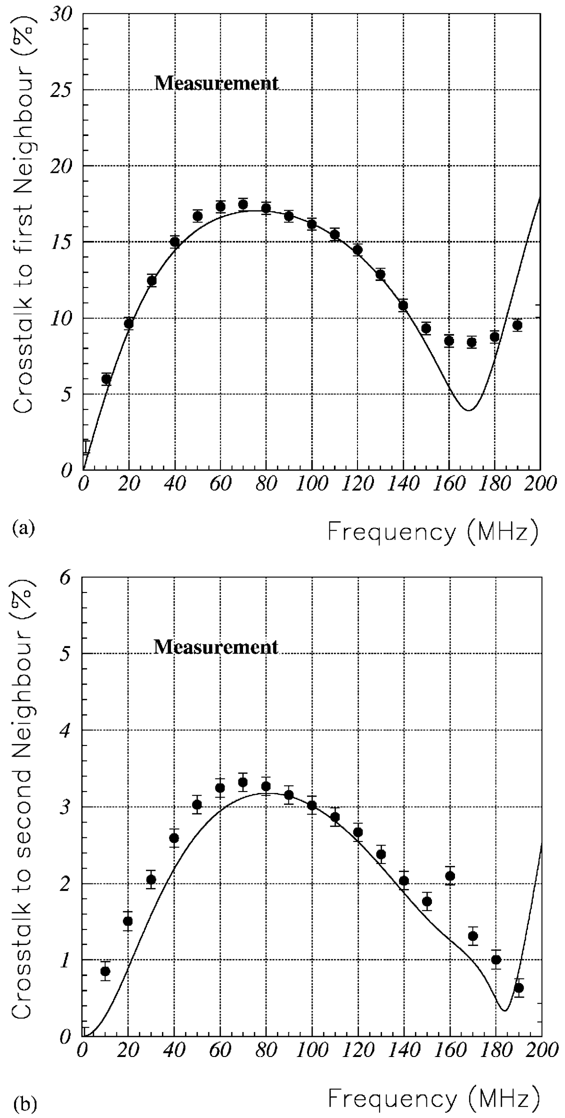

Fig. 16. Measurement of the crosstalk for an RPC with the geometry similar to Fig. 9. A voltage sine wave was put on one strip and the amplitude on the first (a) and second (b) neighbour strip was measured. The Bakelite was assumed to have $\varepsilon = 10$, losses were neglected. The solid line shows the simulation, the points show the measurement. For frequencies < 200 MHz the agreement is acceptable.

with Maxwell) into PSPICE [6]. The agreement is acceptable for frequencies $f < 200$ MHz, which is the frequency range that is important for realistic preamplifiers.

8. Conclusions

We studied signal propagation in RPCs by analyzing the explicit time domain solution of a lossless N-conductor transmission line. Measurements on a RPC prototype show that this model is applicable in the frequency range to which realistic preamplifiers are sensitive. The RPC is completely defined by the 'per unit length' capacitance and inductance matrix that were calculated with Maxwell [4]. The symbolic solution for an induced current pulse $I^0(t)$ at some position $z = z_0$ along the strip is then completely defined and the reflected and measured pulses at the line ends can be calculated with a very elegant matrix formalism. For this report this was done with Mathematica [5].

The formalism allows some general conclusions.

- To avoid reflections on one side of the RPC the strips theoretically have to be interconnected by $\frac{1}{2}N(N+1)$ resistors. The realistic number of interconnections has to be decided depending on specifications.
- Since the RPC is an *inhomogeneous* transmission line the signals propagate as a linear superposition of pulses that are equal to the original induced signal and travel with N different velocities. Therefore we find signal dispersion and dependence of the crosstalk amplitude and shape on preamplifier speed, signal shape and position of the induced signal along the strip.
- The crosstalk is lowest if the preamplifier input resistance is as low as possible, the strips are not interconnected on the preamplifier side and the preamplifiers are as slow as possible. It is therefore important to chose the slowest possible electronics that is still compatible with timing requirements.

Specific to the RPC geometry shown in Fig. 9 we can conclude:

- For a strip length of 2 m the crosstalk to the first neighbour ranges from 3.3% to 13.7% for a preamplifier peaking of 3 ns and from 3.3% to 7.4% for a peaking time of 10 ns ($R_{in} = 25\,\Omega$).

- Since the direct coupling to the second neighbour is very small the crosstalk to the second neighbour happens mainly through the first one and is therefore approximately given by the square of the above numbers. The result is 0.15–1.87% for 3 ns peaking time and 0.15–0.76% for 10 ns peaking time.
- Theoretical considerations show that losses due to conductor resistance, conductivity σ and imaginary permittivity ε_i of the Bakelite should be small within the bandwidth of applicable preamplifiers i.e. <200 MHz.
- Measurements on a prototype confirm that in this frequency range the losses can indeed be neglected. In general however Bakelite is a material that is not very well defined and therefore the losses have to be watched carefully.

Acknowledgements

We would like to thank Giovanni Carboni for providing an RPC and for many useful discussions. We also thank Christoph Posch for many important suggestions.

References

[1] M. Adinolfi, et.al, Proposal for the RPC muon detector of LHCb, LHCb note, LHCb-2000-053, CERN 2000.
[2] ATLAS Muon Spectrometer Technical Design Report, CERN-LHCC-97-22, ATLAS TDR 10, CERN 1997.
[3] C.R. Paul, Analysis of Multiconductor Transmission Lines, Wiley, NewYork, 1994.
[4] MAXWELL 2D Extractor, Version 4.0.05, 1984–1999, Ansoft Corporation.
[5] Mathematica 4.0, Wolfram Research 1999.
[6] PSPICE Analog Circuit Simulator, Version 8.0x, 1998.